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by
Hideya Kato
Mitsuyoshi Yanagihara

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Capital Income Tax Evasion and Welfare Levels in an Overlapping Generations Model

Hideya Kato[a] and Mitsuyoshi Yanagihara[b*]

[a,b] *Graduate School of Economics, Nagoya University, Nagoya, Aichi, 464-8601, Japan*

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Abstract

We construct an overlapping-generations model where individuals evade capital income tax, imposed on their savings and carry out the long-run analysis, as well as the short-run analysis. We consider that the detection probability depends on the amount of undeclared savings, the interest rate and the detection probability parameter.

It is shown that a rise in the capital income tax rate (the detection probability parameter or the penalty rate) lowers (raises) capital stock both in the short- and the long-run. The change in undeclared savings depends on this indirect effect, in addition to the direct effects of such parameters. It is also found that, as opposed to the results of static models, a rise in the tax rate (the detection probability parameter or the penalty rate) can increase welfare in the short-run (the long-run).

Keywords: Tax evasion; Capital income tax; Capital accumulation; Overlapping-generations model

JEL classification: D91; E22; H26

*Corresponding author. Tel.: +81-52-789-5952; fax: +81-52-789-4924; e-mail: yanagi@soec.nagoya-u.ac.jp.

1 Introduction

This paper constructs an overlapping-generations model in which individuals evade a capital income tax imposed on their savings, to examine how the changes in the capital income tax rate, the detection probability of tax evasion and the penalty rate affect the capital stock levels, undeclared savings and the welfare levels of individuals.

As noted, extensive studies treating tax evasion have been carried out from a theoretical standpoint, examining the effects of the actions of governments and tax authorities on the tax evasion behavior of individuals [1]. The first theoretical study of note, which focuses on the tax evasion behavior of individuals, is that of Allingham and Sandmo (1972), where the probability that the concealment of income is detected is given and the amount of the penalty imposed is determined by the level of concealed income [2].

Contrary to this assumption, Yitzhaki (1987) assumes that the probability with which the concealment of income is detected is increasing in the amount concealed. The results obtained in this study are consistent with the facts found by Clotfelter (1983) and Poterba (1987) that a rise in the tax rate will encourage tax evasion behavior. Yitzhaki (1974), on the other hand, presumes that the amount of the penalty depends not on the level of concealed income, but rather on the amount of tax evaded.

In contrast to the above studies which are made within a static and partial equilibrium framework, Caballe and Panades (1997) and Lin and Yang (2001) examine the effects of tax evasion behavior on economic growth from a dynamic macroeconomics viewpoint, within a general equilibrium framework [3]. Caballe and Panades (1997), based on a Diamond

[1] A notable survey on tax evasion theory is that of Cowell (1990) and Andreoni, Brian and Feinstein (1998).

[2] Much of the studies such as Yitzhaki (1974) and Yaniv (1994) have followed Allingham and Sandmo (1972).

[3] Chen (2003) incorporates the tax evasion behavior into Barro (1990) to examine the optimal income

(1965) -type, overlapping-generations model, deals with the tax evasion of a lump-sum tax imposed on the individuals of the young generation, of which penalty is levied during the individual's old period. Under the circumstance that all taxes and penalties are allocated to the public goods which generate a positive externality on the productivity of the private sector, it concludes that a rise in the penalty rate or the detection probability of the concealment of income has a negative effect on "effective savings" (the average level of savings in the economy). Lin and Yang (2001) introduce both a Barro (1990) -type utility function, on which public goods have an externality and an AK-type production function into a portfolio selection model. Contrary to the results obtained by the portfolio selection model in a static framework, a higher income tax rate may encourage tax evasion. It is also revealed that, although a rise in the tax rate leads to an initial drop in economic growth, it will raise economic growth in the long-run.

As shown above, research on the tax evasion behavior of individuals has recently been extended in the direction of taking a dynamic viewpoint. However, past studies have not dealt with tax evasion in terms of savings, i.e., capital income tax evasion, and not infer to the effects on welfare of individuals [4]. As it is well-known, capital income taxation will lower the capital stock level, and therefore, the utility level in the absence of capital income tax evasion; in the presence of capital income tax evasion, it is not clear, though it lowers the capital stock levels, whether the rise in the capital income tax rate necessarily lowers the utility levels of individuals. This is because the change in the capital accumulation, which brings about the changes in the interest rate and wages, will make individuals' tax evasion behavior change in the long-run. As Poterba (1987), which analyzes the effects of marginal tax rates on the reporting of capital gains, emphasizes the importance of a long-

taxation

[4] As stated in the appendix of Clotfelter's (1983) study, while the declaration rate for wage income is 100%, the declaration rate for income from interest and dividend income are 98.3% and 97.5%, respectively.

run effect, as well as a short-run effect, it is required that the capital income tax evasion should be analyzed in a dynamic framework. Therefore, although Caballe and Panades (1997) and Lin and Yang (2001) have all focused on economic growth, the utility levels of the individuals have to be adequately covered in the context of tax evasion behavior.

This paper introduces capital income tax evasion behavior by individuals into the overlapping-generations model by Diamond (1965), and conducts a dynamic analysis that takes capital accumulation into account. In this paper, the detection probability will be endogenized in a Yitzhaki (1987)-fashion. Considering this detection probability and the penalty by the tax authority, the individuals are capable of evading the capital income tax by declaring a false amount of savings, i.e. concealing a part of their savings.

The main results of this paper are as follows: first, on one hand, a rise in the capital income tax rate lowers the level of capital stock in both the short- and the long-run. On the other hand, a rise in the detection probability parameter or the penalty rate increases the levels of capital stock. The latter finding is in contrast to that of Caballe and Panades (1997).

Next, the amount of undeclared savings may increase even when the capital income tax rate falls, or the detection probability parameter or the penalty rate rises. This is because the capital accumulation process is incorporated. That is, the effect on the amount of undeclared savings is determined by the effect on the capital stock, above mentioned.

Lastly, we show that while a rise in the capital income tax rate can increase the welfare, or the utility levels, of individuals in the short-run, a rise in the detection probability parameter or the penalty rate can raise utility in the long-run.

This paper is organized as follows: In Section 2, we will present a basic model. Section 3 examines both the short- and the long-run effects caused by increases in the capital income tax rate, the detection probability and the penalty rate on the levels of capital stock and undeclared savings. In Section 4, we examine the short- and the long-run effects on welfare levels. In Section 5, we give numerical examples. Lastly, Section 6 contains the conclusion of this paper.

2 A Basic Model

The model developed in this paper extends the overlapping-generations model proposed by Diamond (1965) to include circumstances where a capital income tax imposed on individuals can be evaded. The economy begins from an initial period ($t = 1$) and lasts forever. Individuals are identical and live for two periods: the young and the old periods. In every period, there exists the young generation and the old generation. For the sake of simplicity, we assume that there is no population growth and that the population size of each generation is normalized to one.

2.1 Maximization of Individuals' Expected Utility

In this subsection, focusing on the individuals who live in the t-th period as part of the young generation (henceforth called "the t-th generation"), we formulate the optimization behavior of those individuals with an incentive to evade tax.

The individuals supply one unit of labor inelastically to obtain wages, w_t, during their young period. The wages are allocated between consumption during the young period, c_{1t}, and savings, s_t [5]. Therefore, the budget constraint in the young period can be expressed as follows:

$$c_{1t} = w_t - s_t. \tag{1}$$

[5]Subscripts generally indicate the period. However, in regards to the individuals' consumption, the former subscript represents the generation (1 and 2 correspond to the young and the old generation, respectively) while the latter subscript represents the period.

The savings of the individuals are allocated to consumption in the old period. Savings bear interest at a rate of $r_{t+1} > 0$, on which a capital income tax is imposed at a fixed rate, $0 \leq \tau \leq 1$. The net interest rate, which individuals receive, becomes $(1-\tau)r_{t+1}$.

Since the amount of capital income tax is calculated based on the amount of savings as declared by the individuals, it is possible for them to evade the tax by declaring a false amount of savings, i.e., concealing a part of their savings, x_{t+1}. The tax authority can detect this tax evasion behavior by a probability of $0 < p_t < 1$. Following Allingham and Sandmo (1972), when tax evasion is detected, a penalty of the amount $\theta r_{t+1} x_{t+1}$, will be imposed, where $\theta (> \tau)$ is a fixed penalty rate, or a heavy additional tax rate [6]. Therefore, the budget constraint in the old period becomes:

$$\begin{aligned} c_{2t+1} &= (1-p_{t+1})[(1+r_{t+1})s_t - r_{t+1}\tau(s_t - x_{t+1})] \\ &\quad + p_{t+1}[(1+r_{t+1})s_t - r_{t+1}\tau(s_t - x_{t+1}) - \theta r_{t+1} x_{t+1}]. \end{aligned} \tag{2}$$

In order for an incentive to evade tax to exist, the expected return when the tax can be evaded should be larger than that in the case where the tax cannot be evaded, that is, $\tau > p_{t+1}\theta$. In addition, when there exists an inner solution,

$$s_t > x_{t+1} \tag{3}$$

must hold in each period. We assume that the detection probability of tax evasion is increasing in the amount of the individuals' undeclared savings, that is,

$$p_{t+1} = \delta(r_{t+1} x_{t+1})^{\alpha}, \tag{4}$$

[6] Behavior of the government and the tax authority will be covered in detail in Section 2.3.

where $\delta > 0$ is a fixed detection probability parameter which the tax authority can control as a policy variable and $\alpha \geq 1$ [7]. This relation is assumed to be known to all individuals.

Individuals' utility is assumed to be a time-separable log-linear function consisting of the amount of consumption in the young period and the expected amount of consumption in the old period, $u(c_{1t}, c_{2t+1}) = \log(c_{1t}) + \log(c_{2t+1})$ [8]. The individual's problem is to maximize this utility subject to (1), (2) and (4). Then, the amount of undeclared savings and the savings can be obtained as:

$$x_{t+1} = \frac{\Phi}{r_{t+1}}, \tag{5}$$

$$s_t = \frac{w_t}{2} - \frac{\alpha\tau\Phi}{2(1+\alpha)[1+(1-\tau)r_{t+1}]}, \tag{6}$$

where $\Phi \equiv \{\tau/[\delta\theta(1+\alpha)]\}^{1/\alpha} > 0$.

The features of undeclared savings and savings can be stated as follows. First, undeclared savings are decreasing in the interest rate and neutral in regards to the wages. The reason for the former feature is that a rise in the interest rate increases the detection probability of tax evasion. The latter feature is attributed to the fact that the utility function is time-separable.

Second, the savings function is increasing in both the interest rate and the wages. The former feature and be explained as follows. In the absence of tax evasion, it is well-known that, in a two-period setting in which the utility function is a log-linear, the amount of the

[7] This assumption regarding the detection probability is, qualitatively, the same as the one in Yitzhaki (1987), which is $p'(x) > 0$ and $p''(x) \geq 0$.

[8] This formation is the same as the one in Chen (2003).

savings will be independent of the interest rate. However, in the presence of tax evasion, this dose not hold: the change in the interest rate will have an effect on the intertemporal allocation. More concretely, when the interest rate rises, the return from savings increases and therefore, the amount of savings increases. The latter feature is the same one which can be obtained from the log-linearity of the utility function, as the same with the case in the absence of tax evasion.

Summarizing the above results, we arrive at the following Lemma.

Lemma 1 *(1) If individuals' utility function is log-linear and time separable and (2) the detection probability parameter is increasing in the amount of undeclared savings:*

1. *Undeclared savings are decreasing in the interest rate and independent of wages.*

2. *Savings are increasing in the interest rate and increasing in wages.*

Then, we will state the effects of the variables given by the government and the tax authority, τ, δ and θ, on the amounts of savings and undeclared savings. First, when the government raises the capital income tax rate, undeclared savings will increase because the incentive to evade tax rises. On the other hand, the rise in the capital income tax rate decreases the amount of savings because it decreases the return from savings [9].

Second, when the tax authority raises the detection probability parameter or the penalty rate, individuals will increase their savings and decrease their undeclared savings. The return from tax evasion decreases as the detection probability parameter or the penalty rate increases. A decrease in undeclared savings, contrary to the first case, leads to the increase in the amount of savings.

[9] Strictly speaking, there is another negative effect of the rise in capital income tax rate on the amount of savings. As stated, the rise in capital income tax rate increases declared savings, which implies the increase in the return from tax evasion. Therefore, the savings will be less needed.

Summarizing the above results, we arrive at the following Lemma.

Lemma 2 *(1) If individuals' utility function is log-linear and time separable and (2) the detection probability parameter is increasing in the amount of undeclared savings:*

1. *A rise in the capital income tax rate increases undeclared savings and decreases savings.*

2. *A rise in the detection probability parameter or the penalty rate decreases undeclared savings and increases savings.*

As we will see in the following sections, these results may reverse in the general equilibrium framework where capital accumulation is considered. That is, a rise in the capital income tax rate may decrease undeclared savings and a rise in the detection probability parameter or the penalty rate may increase them [10].

2.2 Maximization of Firms' Profit

Firms produce consumption goods from capital and labor, using a Cobb-Douglas type, constant returns to scale production technology. Denoting the capital share and the technology parameter as $0 < \gamma < 1$ and $A > 0$ respectively, produced goods per capita can be expressed as a function of capital stock per capita shown as follows: $y_t = Ak_t^{\gamma}$. Assuming that the price of the goods is normalized to one, that there is no capital depreciation and that production is carried out under a perfect competitive market, the first order conditions for profit maximization can be written as:

$$r_t = A\gamma k_t^{\gamma-1}, \tag{7}$$

[10] See Proposition 1 and 2.

$$w_t = A(1-\gamma)k_t^{\gamma}. \tag{8}$$

No tax is imposed on firms: firms' tax evasion behavior is beyond the scope of this paper.

2.3 Government and Tax Authority Behavior

We have already referred to the taxation behavior of the government, and the tax evasion detection and penalty levy behavior of the tax authority in Section 2.1. In this subsection, we further elaborate on their behavior.

The government imposes a capital income tax at a constant rate on individuals' interest income from savings. The tax authority attempts to detect individuals' tax evasion behavior and imposes a penalty on offenders. We will not explicitly deal with the expenditure side of the government and the tax authority: all government's income, capital income tax and penalty revenue, will be allocated to government consumption and it is assumed that the government keeps a balanced budget on the whole. In addition, any changes to the detection probability parameter is assumed not to incur any costs [11].

2.4 Equilibrium

The capital market equilibrium in each period is attained when individuals' savings (supply of capital) is equal to firms' demand for capital, that is, $k_{t+1} = s_t$ holds. Therefore, from (6), (7) and (8), the capital market equilibrium condition can be expressed as:

$$k_{t+1} = \frac{1}{2}A(1-\gamma)k_t^{\gamma} - \frac{\tau\alpha\Phi}{2(1+\alpha)[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]}. \tag{9}$$

[11] Although many studies assume that a government's income is transferred to the individuals in some way, we will consider the behavior of the government and the tax authority as, so to speak, "given", in order to focus on the individuals' tax evasion behavior.

This condition is clearly dependent upon k_t and k_{t+1}. Then, substituting (7) into (5), the declared savings become

$$x_{t+1} = \frac{\Phi}{A\gamma k_{t+1}^{\gamma-1}}. \tag{10}$$

Similarly, in the steady state, the capital market equilibrium can be rewritten by substituting $k_t = k_{t+1} = k$ into (9):

$$k = \frac{1}{2}A(1-\gamma)k^{\gamma} - \frac{\tau\alpha\Phi}{2(1+\alpha)[1+(1-\tau)A\gamma k^{\gamma-1}]}, \tag{11}$$

and the declared savings can be rewritten as

$$x = \frac{\Phi}{A\gamma k^{\gamma-1}}. \tag{12}$$

3 The Effects of Changes to the Policy Parameters on Capital Stock

In this section, first, we will examine the short- and the long-run effects on the capital stock level and undeclared savings when the government raises the capital income tax rate for period $t+1$. Next, we will examine the effects caused when the tax authority raises the detection probability parameter or the penalty rate. It must be noted that we assume that once the policy parameters have been changed, they will be kept constant.

3.1 The Effects of the Capital Income Tax

As seen in Appendix, when the government raises the capital income tax rate, the short- and the long-run effects on the capital stock are as follows:

$$\frac{dk_i}{d\tau} = -\frac{\Phi}{\Delta_i}\frac{1+\alpha+(1+\alpha-\tau)A\gamma k_i^{\gamma-1}}{2(1+\alpha)[1+(1-\tau)A\gamma k_i^{\gamma-1}]^2} < 0, \tag{13}$$

where $\Delta_i > 0$, and $i = t+1$ in the short-run and $i = s$ in the long-run (the steady state) [12]. From (13), a rise in the capital income tax rate decreases the capital stock in both the short- and the long-run.

A rise in the tax rate will have two effects on savings incentive. One, a negative savings incentive effect: a rise in the tax rate will discourage the incentive to save, and decrease the capital stock. In the short-run, being as this is the only effect present, the capital stock will always decrease. Two, a long-run savings incentive effect: in addition to the negative effect, the above mentioned decrease in the capital stock (or the supply of capital) will raise the interest rate, which will in turn lower wages in the long-run. It should be noted that a rise in the interest rate will attract savings, and hence, work in the direction of raising the capital stock. Therefore, the long-run savings incentive effect works in either direction. However, since the first negative effect dominates the second effect, the capital stock will decrease in both the short-and the long-run.

On the other hand, the effects on undeclared savings are as follows:

$$\frac{dx_i}{d\tau} = \frac{\Phi}{\alpha\tau A\gamma k_i^{\gamma-1}} + \frac{(1-\gamma)\Phi}{A\gamma k_i^{\gamma}}\frac{dk_i}{d\tau}. \tag{14}$$

[12] Henceforth, we will use these expressions, unless otherwise noted.

While the first term of (14) is positive, as indicated in (13), the second term becomes negative. Therefore, neither the signs in the short- nor the long-run is determined on the whole. These terms can be interpreted as having two separate effects on undeclared savings, the same as the aforementioned effect on the capital stock. One, the first term is a (direct) positive undeclared savings incentive effect, as shown in Lemma 2: a rise in the tax rate stimulates the incentive to evade tax. Two, the second term is a (indirect) negative undeclared savings incentive effect. As we have seen in (13), a rise in the capital income tax rate lowers the capital stock. This brings about an increase in the interest rate, and subsequently, a rise in the detection probability. Consequently, as mentioned in Lemma 1, undeclared savings will decrease.

In general, because the direct effect will dominate the indirect effect, a rise in the capital income tax rate has the effect of increasing undeclared savings. Oppositely, if the latter dominates the former, the overall effect will prove to be negative. That is, the direction of the effect is determined by the magnitude of the above effects on the capital stock.

The results obtained here can be summarized in the following Proposition.

Proposition 1 *When the capital income tax rate rises, in both the short- and the long-run,*

1. *The capital stock decreases.*
2. *Undeclared savings decrease if the negative indirect effect dominates the positive direct effect.*

3.2 The Effects of Detection Probability Parameter and the Penalty Rate

In this subsection, we will examine the effects of a rise in the detection probability parameter or the penalty rate by the tax authority on the capital stock and undeclared savings.

The effects of rises in the detection probability parameter and the penalty rate on the capital stock are

$$\frac{dk_i}{d\delta} = \frac{1}{\Delta_i} \frac{\tau\Phi}{2(1+\alpha)\delta[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]} > 0, \tag{15}$$

$$\frac{dk_i}{d\theta} = \frac{1}{\Delta_i} \frac{\tau\Phi}{2(1+\alpha)\theta[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]} > 0, \tag{16}$$

respectively. In both cases, these signs are positive (in both the short- and the long-run).

These results can be divided into two effects. The first effect is a positive direct effect on savings (and capital stock): an increase in these parameters will induce an increase in the return from savings as shown in Lemma 2. As this is the only effect present in the short-run, the capital stock will necessarily increase. The second effect is an indirect effect brought on by a change in the interest rate, which we have seen in the previous section. Since the former effect dominates the latter effect, the capital stock will necessarily increase in both the short- and the long-run.

Based on the above results, the effects of an increase in the detection probability parameter or the penalty rate on undeclared savings can be written as follows:

$$\frac{dx_i}{d\delta} = -\frac{\Phi}{\alpha\delta A\gamma k_i^{\gamma-1}} + \frac{(1-\gamma)\Phi}{A\gamma k_i^{\gamma}}\frac{dk_i}{d\delta}, \tag{17}$$

$$\frac{dx_i}{d\theta} = -\frac{\Phi}{\alpha\theta A\gamma k_i^{\gamma-1}} + \frac{(1-\gamma)\Phi}{A\gamma k_i^{\gamma}}\frac{dk_i}{d\theta}. \tag{18}$$

While the first terms are always negative, as seen in (15) and (16), the second terms are always positive. As such, these signs are not decisive. The first negative effect is to lower the incentive for tax evasion directly and the second positive effect is to stimulate the incentive, as a result of a fall in the interest rate caused by an increase in the capital stock [13].

In general, as it is the case with a rise in the capital income tax rate, since the direct effect generally dominates the indirect effect, a rise in the detection probability parameter or the penalty rate tends to decrease undeclared savings. In contrast, if the effect on the capital stock is sufficiently large enough that the indirect effect dominates the direct effect, a rise in the policy parameters *increases* undeclared savings.

These results can be summarized as follows.

Proposition 2 *When the detection probability parameter or the penalty rate rises, in both the short- and the long-run:*

1. *The capital stock increases.*
2. *Undeclared savings increase if the positive indirect effect dominates the negative direct effect.*

[13] It should be noted that the detection probability is dependent upon the interest rate

4 The Effects of Changes to the Policy Parameters on Utility Levels

To see the effects of a rise in the capital income tax rate, the detection probability parameter or the penalty rate on the utility levels of individuals, we first describe the utility levels of individuals in the short-run can be written as:

$$\begin{aligned} v(k_t, k_{t+1}; \tau, \theta, \delta) &= \log[A(1-\gamma)k_t^{\gamma} - k_{t+1}] \\ &\quad + \log\left[k_{t+1} + (1-\tau)A\gamma k_{t+1}^{\gamma} + \frac{\alpha\tau}{1+\alpha}\Phi\right]. \end{aligned} \tag{19}$$

In the same way, the utility levels of individuals in the long-run can be written as:

$$\begin{aligned} v(k; \tau, \theta, \delta) &= \log[A(1-\gamma)k^{\gamma} - k] \\ &\quad + \log\left[k + (1-\tau)A\gamma k^{\gamma} + \frac{\alpha\tau}{1+\alpha}\Phi\right]. \end{aligned} \tag{20}$$

4.1 The Effects of the Capital Income Tax Rate on Utility Levels

First, differentiating (19) with respect to τ, we can obtain the effect of the capital income tax rate on utility levels in the short-run,

$$\frac{dv(k_t, k_{t+1}; \tau, \theta, \delta)}{d\tau} = \frac{A\gamma k_{t+1}^{\gamma-1}}{c_{2t+1}}\left\{-(k_{t+1} - x_{t+1}) - (1-\tau)(1-\gamma)\frac{dk_{t+1}}{d\tau}\right\}, \tag{21}$$

where all values of the variables are the ones obtained in the former section. From (3), (10) and $k_{t+1} = s_t$, the sign of the first term is negative, and from (13), the sign of the second term is positive [14]. The first negative effect is that the rise in the capital income tax rate directly decreases utility levels, because it reduces the return from declared savings. The second positive effect is that the rise in the capital income tax rate indirectly increases utility levels, because the decrease in capital stock raises the interest rate, and in turn, raises return from savings. In conclusion, the effect of the capital income tax rate on utility levels in the short-run depends on the direct and the indirect effects, and therefore, the sign of (21) is indeterminate.

Next, differentiating (20) with respect to τ, the effect of the capital income tax rate on utility levels in the long-run can be obtained by:

$$\frac{dv(k;\tau,\theta,\delta)}{d\tau} = \frac{A\gamma k^{\gamma-1}}{c_{2s}} \left\{ -(k-x) + (1-\gamma)[1-(1-\tau)(1-A\gamma k^{\gamma-1})]\frac{\partial k}{\partial \tau} \right\} < 0. \quad (22)$$

It is clear that the sign is always negative, that is, the rise of the capital income tax rate unambiguously decreases utility in the long-run. This result can be also interpreted into two effects, the negative direct effect and indirect effects. The direct effect, which is brought by the decrease in the return from declared savings, is represented by the first term; the indirect effect, which is caused by the decrease in the return from savings, is represented by the second term. Unlike the case of the short-run, the decrease in capital stock lowers utility levels in the long-run, because the economy is always dynamically efficient.

The results obtained here can be summarized in the following Proposition.

[14] It should be noted that because $x_{t+1} = \Phi/(A\gamma k_{t+1}^{\gamma-1})$ and $k_{t+1} = s_t$, the first term in the curly brackets represents the "declared savings", $(s_t - x_{t+1})$.

Proposition 3 *When the capital income tax rate rises,*

1. *Utility levels always fall in the long-run.*
2. *Utility levels rise in the short-run if the positive indirect effect dominates the negative direct effect.*

4.2 The Effect of the Detection Probability Parameter and the Penalty Rate on Utility Levels

First, differentiating (19) with respect to δ and θ, we can obtain the effects of such parameters as the detection probability parameter and the penalty rate on utility levels in the short-run:

$$\frac{dv(k_i, k_j; \tau, \theta, \delta)}{d\delta} = -\frac{\theta E}{c_{2j}} - \frac{(1-\tau)(1-\gamma)}{c_{2t+1}}\frac{\partial k_j}{\partial \delta} < 0, \tag{23}$$

$$\frac{dv(k_i, k_j; \tau, \theta, \delta)}{d\theta} = -\frac{\delta \Phi^{1-\alpha}}{c_{2j}} - \frac{(1-\tau)(1-\gamma)}{c_{2j}}\frac{\partial k_j}{\partial \theta} < 0, \tag{24}$$

where the signs of (23) and (24) are always negative.

The effects can be interpreted into two negative effects: the direct and the indirect effects. The first effect is brought by the decrease in the return from declared savings; the second effect is caused by the decrease in the return from savings, because the increase in capital stock decreases the interest rate.

Next, the long-run effects are calculated as:

$$\frac{dv(k;\tau,\theta,\delta)}{d\delta} = -\frac{\theta E}{c_{2s}} + \frac{(1-\gamma)A\gamma k^{\gamma-1}[1-(1-\tau)(1-A\gamma k^{\gamma-1})]}{c_{2s}}\frac{\partial k}{\partial \delta}, \tag{25}$$

$$\frac{dv(k;\tau,\theta,\delta)}{d\theta} = -\frac{\delta E}{c_{2s}} + \frac{(1-\gamma)A\gamma k^{\gamma-1}[1-(1-\tau)(1-A\gamma k^{\gamma-1})]}{c_{2s}}\frac{\partial k}{\partial \theta}. \tag{26}$$

The signs of the first term in (25) and (26) is negative and the sign of the second term is positive. Hence, the signs of (25) and (26) become ambiguous, that is, the rise in these parameters are independent upon the configuration of these direction and indirect effects. The direct effect is brought by the decrease in the return from undeclared savings. On the other hand, the indirect effect is from the increase in capital stock.

The results obtained here can be stated in the following Proposition.

Proposition 4 *When the detection probability parameter or the penalty rate rises,*

1. *Utility levels always fall in the short-run.*
2. *Utility levels rise in the long-run if the positive indirect effect dominates the negative direct effect.*

In Table 1 and 2, we summarize the results obtained in Lemma 2 under the subjective equilibrium and in Proposition 1 to 4 under the general equilibrium.

5 Numerical Examples

In this section, we will present numerical examples for a comparative static analysis, that is, the effects of γ, A, τ, θ and δ on k, x and v, so as to clarify that the results mentioned above might occur. Here, we use two types of capital share. The one is that in a "narrow sense," in which capital is defined only as physical capital, is about 0.3. The other one is that in a "broad sense," in which capital is defined not only as physical but human capital. For example, it is 0.75 in Barro and Sala-i-Martin (1995) and is from 0.35 to 0.6 in Mankiw, Romer and Weil (1992). Therefore, we use the capital share in its "narrow sense" in Table 3 and 4 ($\gamma = 0.3$), and the one in its "broad sense" in Table 5 ($\gamma = 0.6$), respectively.

For each table, the second column contains the given (initial) values of τ, θ and δ, and the third to fifth columns contain the (initial) equilibrium values of the capital stock, k, undeclared savings, x, and utility level, v respectively. The sixth column indicates the condition of dynamic stability, the value of dk_{t+1}/dk_t (the upper figure), and the detection probability, p (the lower figure). If $0 < \Delta < 1$ holds, the stability condition is satisfied; this condition holds for all cases given in the Table. In addition, the detection probability ranges from 0 to 1 for all cases, which is consistent with the presumption. The results of the comparative static analysis are shown in the eighth to thirteenth columns: where the parameters in the seventh column are changed, the short-run effects of k, x and v are written in the former three columns, and the long-run effects are in the latter three, respectively.

Table 3 treats an "orthodox" case, as we have indicated in Proposition 1 to 4, where, in general, a rise in τ increases x and decreases v, and a rise in θ or δ decreases both x and v.

On one hand, though the direction of change in the capital stock is the same in both the short- and the long-run, the degree of change in the long-run is larger than that in the short-run: by raising τ, the capital stock decreases more and by raising θ and δ, the capital stock will decreases less in the long-run. This is expected because monotonous convergence to the steady state is assumed. On the other hand, in regards to undeclared savings, the long-run effects are smaller than the short-run effects.

In regards to the effects on utility levels, a rise in τ lowers them in the long-run more than in the short-run; in contrast, the reverse is true for a rise in θ and δ. These findings correspond to the changes in the capital stock discussed above: as noted in Proposition 1, whether in the short-run or the long-run, the capital stock level will decrease. To elaborate, in the case of a rise in τ, the level of capital stock in the long-run becomes lower than that in the short-run, while in the case of a rise in θ and δ, the level of capital stock in the long-run becomes higher than that in the short-run; therefore, in the long-run, the resources in the economy will decrease and increase, respectively [15].

Table 4 and 5 show two “paradoxical” cases with regards to the effects on utility levels, as we have mentioned in Proposition 3 and 4: utility levels become higher in the short-run when τ rises (Table 4), and they become higher in the long-run when θ and δ rise (Table 5). In the former case, because δ is low enough that individuals can evade tax without difficulty, it can be thought that a rise in the capital income tax rate increases undeclared savings; accordingly, (the expected) consumption in the old period also increases. In the latter case, it can be recognized that the indirect effect caused by capital accumulation dominates the direct effect [16]. In this case, it must be noted that, the larger the capital share of the production function, the more highly accumulated capital will be in the long-

[15] This result can be applied to all of the following cases.

[16] While the indirect effect on the short-run utility level is brought about by a change in the interest rate, the effect on the long-run utility level and the change in the interest rate are brought about by a change in wage levels. Also refer to the argument in Section 2.1.

run.

6 Conclusion

We have analyzed the capital income tax evasion behavior of individuals from a dynamic viewpoint as opposed to a static one, in order to illustrate the long-run effects of changes of the policy parameters from the standpoint of capital accumulation. We have also incorporated tax evasion behavior into a general equilibrium framework. This makes it possible to investigate the effects of changes of the policy parameters on the economy as a whole, which could not have been examined in a partial equilibrium framework.

The main results obtained in this paper are as follows. First, while a rise in the capital income tax rate decreases the levels of capital stock in both the short- and the long-run, a rise in the detection probability parameter or the penalty rate increases the levels of capital stock in both the short- and the long-run. This difference results from how the incentive to save is affected. In the former case a rise in the capital income tax rate discourages the incentive to save. Meanwhile, in the latter case, a rise in the policy parameters decreases the expected return from savings, thereby stimulating the incentive to save.

Second, in regards to the effects on undeclared savings, the magnitude of the effects on the capital stock works as a determinant factor, in all cases where the capital income tax rate, the detection probability parameter or the penalty rate rises. More specifically, while a rise in the capital income tax rate has the direct effect of increasing undeclared savings, since a fall in the (total amount of) savings will bring about an increase in the interest rate, the indirect effect will work in the direction of decreasing undeclared savings. Conversely, when the detection probability parameter or the penalty rate rises, the opposite effects will occur.

Third, for the effects of changes to the policy parameters on welfare, or utility levels, a rise in the capital income tax rate may have a positive effect on welfare in the short-run when tax evasion behavior is taken into consideration. Similarly, a rise in the detection probability parameter or the penalty rate may bring about a positive effect on welfare in the long-run. These results imply that policy tools that discourage tax evasion behavior (decrease the amount of undeclared savings) are, under certain conditions, preferable to individuals' welfare.

Although this dynamic general equilibrium model has succeeded in presenting some economic implications, our simplification of government and tax authority behavior may have been excessive. One possibility for modification is to adopt an approach that takes the expenditure side into consideration. In our analysis, all of the income collected by the government and the tax authority is assumed to be disposed of, or consumed. Some correction will be required when considering cases where the income is to be transferred to individuals in a lump-sum fashion, or allocated to the public goods supply. Another possibility for modification is to endogenize the behavior of the government and tax authority. Also, it may be more plausible to assume that the government is capable of carrying out optimal taxation. Lastly, strategic settings, or game theoretical frameworks could be introduced into individual and government (or tax authority) behavior.

A Appendix

This appendix derives the results of the comparative static analysis in Section 3. First, totally differentiating (9) we find that:

$$
\begin{aligned}
\Delta_{t+1} dk_{t+1} =& \Delta_t dk_t - \frac{1+\alpha+(1+\alpha-\tau)A\gamma k_{t+1}^{\gamma-1}}{2(1+\alpha)[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]^2}\Phi d\tau \\
&+ \frac{\tau\Phi}{2(1+\alpha)\delta[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]} d\delta \\
&+ \frac{\tau\Phi}{2(1+\alpha)\theta[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]} d\theta,
\end{aligned} \tag{27}
$$

where, $\Delta_t \equiv A\gamma(1-\gamma)k_t^{\gamma-1}/2 > 0$, $\Delta_{t+1} \equiv 1 + \frac{\tau(1-\tau)\alpha A\gamma(1-\gamma)k_{t+1}^{\gamma-1}}{2(1+\alpha)[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]}\left[\frac{\tau}{\delta\theta(1+\alpha)}\right]^{1/\alpha} > 0$.

We evaluate (27) in the steady state:

$$
\begin{aligned}
\Delta_s dk =& - \frac{1+\alpha+(1+\alpha-\tau)A\gamma k_{t+1}^{\gamma-1}}{2(1+\alpha)[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]^2}\Phi d\tau \\
&+ \frac{\tau\Phi}{2(1+\alpha)\delta[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]} d\delta \\
&+ \frac{\tau\Phi}{2(1+\alpha)\theta[1+(1-\tau)A\gamma k_{t+1}^{\gamma-1}]} d\theta,
\end{aligned} \tag{28}
$$

where Δ_s is the value of $\Delta \equiv \Delta_{t+1} - \Delta_t$ evaluated in the steady state, and k is the capital stock in the steady state. Furthermore, the condition for monotonic convergence to the steady state is:

$$
\begin{aligned}
0 < \frac{dk_{t+1}}{dk_t} =& \frac{\Delta_t}{\Delta_{t+1}} < 1 \\
&\iff \Delta > 0.
\end{aligned} \tag{29}
$$

Under these conditions, the results of the comparative static analysis in the steady state can be obtained from (28). The short-run effects in the comparative static analysis can also be obtained from (27) in the same manner: by substituting Δ_s with Δ_{t+1}.

Next, totally differentiating (10), we arrive at the following:

$$\begin{aligned} dx_{t+1} &= \left\{ \frac{\Phi}{\alpha\tau A\gamma k_{t+1}^{\gamma-1}} + \frac{(1-\gamma)\Phi}{A\gamma k_{t+1}^{\gamma}} \frac{dk_{t+1}}{d\tau} \right\} d\tau \\ &+ \left\{ -\frac{\Phi}{\alpha\delta A\gamma k_{t+1}^{\gamma-1}} + \frac{(1-\gamma)\Phi}{A\gamma k_{t+1}^{\gamma}} \frac{dk_{t+1}}{d\delta} \right\} d\delta \\ &+ \left\{ -\frac{\Phi}{\alpha\theta A\gamma k_{t+1}^{\gamma-1}} + \frac{(1-\gamma)\Phi}{A\gamma k_{t+1}^{\gamma}} \frac{dk_{t+1}}{d\theta} \right\} d\theta. \end{aligned} \tag{30}$$

Evaluating (30) in the steady states gives:

$$\begin{aligned} dx &= \left\{ \frac{\Phi}{\alpha\tau A\gamma k^{\gamma-1}} + \frac{(1-\gamma)\Phi}{A\gamma k^{\gamma}} \frac{dk}{d\tau} \right\} d\tau \\ &+ \left\{ -\frac{\Phi}{\alpha\delta A\gamma k^{\gamma-1}} + \frac{(1-\gamma)\Phi}{A\gamma k^{\gamma}} \frac{dk}{d\delta} \right\} d\delta \\ &+ \left\{ -\frac{\Phi}{\alpha\theta A\gamma k^{\gamma-1}} + \frac{(1-\gamma)\Phi}{A\gamma k^{\gamma}} \frac{dk}{d\theta} \right\} d\theta. \end{aligned} \tag{31}$$

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Table 1: Comparative Statics

	Lemma 2		Proposition 1	Proposition 2
	τ	δ, θ	τ	δ,θ
k (s)	$-$	$+$	$-$	$+$
x	$+$	$-$	$*1$	$*2$

$*1$: This sign becomes negative (positive), if the negative indirect effect dominates (is dominated by) the positive direct effect.

$*2$: This sign becomes positive (negative), if the positive indirect effect dominates (is dominated by) the negative direct effect.

Table 2: The Effects on the Utility Levels

	Proposition 3		Proposition 4	
	τ		δ, θ	
	Short	Long	Short	Long
v	$*3$	$-$	$-$	$*4$

$*3$: This sign becomes positive (negative), if the positive indirect effect dominates (is dominated by) the negative direct effect.

$*4$: This sign becomes positive (negative), if the positive indirect effect dominates (is dominated by) the negative direct effect.

Table 3: Comparative Statics (Orthodox Case) : $\gamma = 0.3, A = 6.0$

case	initial	equilibrium			Δ	chg.	short-run			long-run		
No.	value	k	x	v	/p		k	x	v	k	x	v
	$\tau = 0.2$					τ	-0.34	6.30	-0.24	-0.49	6.25	-0.33
1-1	$\theta = 0.3$	2.84	1.28	2.66	0.70	θ	0.11	-4.24	-0.09	0.16	-4.22	-0.06
	$\delta = 0.3$				/0.33	δ	0.11	-4.24	-0.09	0.16	-4.22	-0.06
	$\tau = 0.2$					τ	-0.26	4.76	-0.31	-0.37	4.73	-0.37
1-2	$\theta = 0.4$	2.85	0.96	2.66	0.70	θ	0.06	-2.40	-0.05	0.09	-2.39	-0.03
	$\delta = 0.3$				/0.25	δ	0.08	-3.19	-0.06	0.12	-3.19	-0.04

Table 4: Comparative Statics ($\tau \uparrow \Longrightarrow$ Short-run $v \uparrow$) : $\gamma = 0.3, A = 8.0$

case	initial	equilibrium			Δ	chg.	short-run			long-run		
No.	value	k	x	v			k	x	v	k	x	v
	$\tau = 0.2$					τ	-1.02	18.36	0.02	-1.47	18.07	-0.16
2	$\theta = 0.3$	4.21	3.80	3.50	0.70	θ	0.32	-12.46	-0.17	0.47	-12.37	-0.11
	$\delta = 0.1$				/0.33	δ	0.97	-37.39	-0.51	1.40	-37.12	-0.34

Table 5: Comparative Statics (θ,$\delta \uparrow \Longrightarrow$ Long-run $v \uparrow$) : $\gamma = 0.6, A = 6.0$

case	initial	equilibrium			Δ	chg.	short-run			long-run		
No.	value	k	x	v			k	x	v	k	x	v
	$\tau = 0.2$					τ	-0.18	1.82	-0.63	-0.45	1.79	-0.92
3	$\theta = 0.3$	1.54	0.37	2.13	0.40	θ	0.05	-1.22	-0.08	0.14	-1.21	0.01
	$\delta = 0.3$				/0.33	δ	0.05	-1.22	-0.08	0.14	-1.21	0.01