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by  
**Hikaru Ogawa**

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# Tax Competition, Spillovers, and Subsidies

Hikaru Ogawa\*

## Abstract

In the literature of local public finance, one of the well-known property of optimal matching grant program is that the matching grant rate should increase as the degree of benefit spillovers of public goods increases. This paper presents the re-examination of properties of optimal matching grant program, using the model of Bjorvatn and Schjelderup (2002). The result formally captures a quite counter-intuitive property of matching grant that optimal matching grant rate might decrease with the degree of spillover externality.

[keywords] tax competition, spillover externality, matching grant.

[JEL classification code] H4, H7, R5

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\* Associate Professor, School of Economics, Nagoya University,  
email: ogawa@soec.nagoya-u.ac.jp

# 1 Introduction

Traditional analyses had suggested that a benefit spillover of local public goods would cause local governments to provide suboptimal level of public goods. Local provision of public goods generating spillover externality should be enhanced by a Pigovian subsidies made by the central government [see Williams (1966), Pauly (1970), and Oates (1972) among others].

The important implication of optimal subsidy policy derived in the literature is that as the degree of spillovers increases, the optimal matching grant rate increases. In the context of regional governments within a country, benefit spillover of public goods is an inevitable phenomenon since the jurisdictional boundaries do not coincide with the range that the benefit spreads to. In the international context, looking at the development of global environmental issue as a simple example, we find that a correlation among the national governments' activities is fairly strengthened compared with the past in the world economy. Moreover, political efforts to create broad economic unions is another recognition of the importance of international spillovers. The integration such as European Union must have promoted the diffusion of benefit originated from each country's public service.

Facing these situations, we find from conventional argument that we need an extensive subsidy program that would be implemented by supra-regional government. Many studies, including King (1984), Lee (1995), Lockwood (1999), Figuières and Hindriks (2002), Akai and Ihuri (2002) among others follow the conventional argument and use the matching grant to stimulate regional public goods provision.

The purpose of this paper is to provide the reexamination of the properties of optimal matching grant program. While most of the traditional models are highly simplified to obtain clear-cut results, this paper extends their studies to incorporate some realistic factors into the model, focusing

on the production aspect. More specifically, this paper incorporates the production sector, capital mobility, and distorting taxation explicitly into the model. This generalization leads our analytical framework to the tax competition model with spillover externality developed by Wildasin (1991) and Bjorvatn and Schjelderup (2002). Specifically, this paper is closely related to Bjorvatn and Schjelderup's paper which derives a result that there arises no fiscal externality problem when the benefits of local public goods perfectly spillout to the other regions. Using the model of Bjorvatn and Schjelderup (2002), this paper formally determines the optimal matching rate to reexamine the property of optimal matching grant policy.

By this extension of the model, somewhat counter-intuitive result is obtained for the effect of changes in the degree of benefit spillovers of public goods on the optimal matching grant policy. This paper shows that the conventional argument regarding the effect of increase in the degree of benefit spillover on the matching grant rate might not hold. In this paper, by introducing the factor mobility into the model, there arises another source of inefficiency, called fiscal externality. The existence of fiscal externality acts on the direction that regional governments underprovide public goods. Then, we argue that the increase in spillover actually reduces the distortion due to the fiscal externality, so that optimal matching grant rate might decrease.

In the next section, we redevelop a basic spillover model in the absence of a production sector. In section 3, the production sector and factor mobility are introduced into the basic model to show that there are reasons to consider that an optimal matching grant rate should be reduced as the degree of benefit spillovers of public goods increases.

## 2 Basic Model

We start from the basic model used in Oates (1972), Lee (1995), and Boadway et al. (1989). There are  $n$  identical regions, and in each region  $i$ , there is a single immobile resident, with strictly quasi-concave preferences  $u(x_i, G_i)$  defined over consumption of a private numeraire good  $x$  and a public good  $G$ . The assumption of identical regions allows us to isolate the efficiency problem of public good provision from the equity issues. The public good consumption in  $i$  is defined by

$$G_i \equiv g_i + \beta \sum_{j \neq i} g_j, \quad (1)$$

where  $g_i$  is the provision of public good by regional government  $i$ , and  $\beta \in (0,1]$  is the degree of benefit spillover.

We initially assume that each region is endowed with an exogenous income  $y_i$ , and the governments impose lump-sum taxes on the resident. The resident's budget constraint will be given by

$$x_i = y_i - \tau_i - z_i, \quad (2)$$

where  $\tau_i$  and  $z_i$  are the taxes imposed by the regional and central governments, respectively. The regional government  $i$  raises revenue  $\tau_i$  and receives a grant from the central government,

$$s_i = m_i g_i, \quad (3)$$

where  $m_i$  is the rate of the matching grant. Fiscal revenue serves to finance the provision of local public goods. The private goods can be used as an input to produce local public goods, and units can be chosen so that the public good provision in region  $i$  can be measured in terms of units of private

goods. Hence, the budget constraint of the regional government can be given by

$$\tau_i + s_i = g_i. \quad (4)$$

$z_i$  in (2) will be chosen so as to satisfy the budget constraint of the central government,  $\sum_i m_i g_i = \sum_i z_i$ .

The regional government  $i$  wishes to maximize the utility of its resident subject to (1)-(4) taking the tax rates of other regions as given. Then, the maximization problem will be defined as

$$\max_{\tau_i} u^i = u \left( y_i - \tau_i - z_i, \frac{\tau_i}{1 - m_i} + \beta \sum_{j \neq i} \frac{\tau_j}{1 - m_j} \right)$$

The first-order condition gives us

$$u_g^i = u_x^i (1 - m_i). \quad (5)$$

This condition can be compared with the Pareto optimal condition,

$$u_g^i + \beta \sum_{j \neq i} u_g^j = u_x^i, \quad (6)$$

which is derived by the maximization of  $\sum_i u^i$  subject to the national resource constraint,  $\sum_i x_i + \sum_i g_i = \sum_i y_i$ . We now assume that the rates of the matching grant are the same for all regions<sup>1</sup>. Then, since we have considered the case of identical regions, (5) and (6) can be, respectively, rewritten as

$$\frac{u_g}{u_x} = 1 - m \quad (7)$$

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<sup>1</sup>As shown in Lee (1995), there exists at least one policy combination of  $(m, z_i)$  that is Pareto improving. We can easily show that there is a large number of efficient resource allocations if we allow the grant rate vary across the regions. See Boadway et al. (1989).

and

$$\frac{u_g}{u_x} = \frac{1}{1 + \beta(n - 1)}, \quad (8)$$

where the region specific subscript  $i$  is omitted. Comparison of (7) and (8) shows that an optimal rate of the matching grant is simply given by

$$m = \frac{\beta(n - 1)}{1 + \beta(n - 1)}, \quad (9)$$

which reproduces the following well known result.

Proposition 1.  $\partial m / \partial \beta > 0$  and  $\partial m / \partial n > 0$ . That is, the optimal matching grant rate increases with the degree of spillovers and the number of regions.

### 3 Tax Competition and Spillovers

So far, we have assumed that the regional income is exogenously given and governments are allowed to use a lump-sum tax. In this section, we now introduce the production sector and factor (capital) mobility explicitly and assume regional government can raise revenue only with the distortional capital tax. This causes so-called tax competition among the regions. The basic tax competition model we use to derive the optimal matching grant policy here is the same form as that used in Wildasin (1988,1991), Hoyt (1991), Bjorvatn and Schjelderup (2002)<sup>2</sup>.

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<sup>2</sup>In the capital tax competition literature, an optimal subsidy program to correct an inefficient resource allocation has been studied previously by Wildasin (1989) and DeParter and Myers (1994). The framework used in this paper is similar in some respects to those studies, but their interests are not on the effects of changes in  $\beta$  and  $n$  on the optimal subsidy rate.

Production of private goods in region  $i$  is conducted by a large number of identical firms and requires using capital and labor. The aggregate production function in region  $i$  is  $f(k_i)$  ( $f_k(k_i) > 0, f_{kk}(k_i) < 0$ ), where  $k_i$  is the amount of capital located in region  $i$  and labor is omitted. The economy has a fixed stock of capital,  $\bar{k}$ , which is perfectly mobile among regions. In the equilibrium, therefore, the net (after tax) return to capital will be equalized across the regions,

$$f_k(k_i) - t_i = \rho, \quad \forall i \quad (10)$$

where  $t_i$  is the capital tax rate and  $\rho$  is the net return to capital. The total supply of capital is fixed at  $\bar{k}$  such that

$$\bar{k} = \sum_{i=1}^n k_i. \quad (11)$$

The resident in region  $i$  receives labor income,  $f(k_i) - f_k(k_i)k_i$ , and the rent from capital,  $\rho\bar{k}_i$ , where  $\bar{k}_i$  is the initial endowment of capital in region  $i$ . Hence, the budget constraint of the resident requires  $x_i = f(k_i) - f_k(k_i)k_i + \rho\bar{k}_i - z_i$ , which can be rewritten as

$$x_i = f(k_i) - t_i k_i + \rho(\bar{k}_i - k_i) - z_i, \quad (12)$$

by using (10).

Now, the local public good in region  $i$ ,  $g_i$ , is financed by capital taxation,  $t_i k_i$ , and the subsidy from the central government,

$$s_i = m_i g_i. \quad (13)$$

Therefore, the regional government budget constraint is given by

$$t_i k_i + s_i = g_i \quad (14)$$



Substituting (12), (13) and (14) into the resident's utility function, the maximization problem of the regional government  $i$  is now defined as

$$\max_{t_i} u_i = u \left( f(k_i) - t_i k_i + \rho(\bar{k}_i - k_i) - z_i, \frac{t_i k_i}{1-m} + \beta \sum_{j \neq i} \frac{t_j k_j}{1-m} \right),$$

where we have already assumed that the matching grant rates are the same for all regions. Assuming that regional governments act under the Nash assumption, the tax rate will be implicitly solved by

$$\frac{u_g^i}{u_x^i} = \frac{(1-m)[k_i - \frac{\partial \rho}{\partial t_i}(\bar{k}_i - k_i)]}{k_i \left(1 + \frac{\partial k_i}{\partial t_i} \frac{t_i}{k_i}\right) + \beta \sum_{j \neq i} t_j \frac{\partial k_j}{\partial t_i}} \quad (15)$$

Since we have assumed that regions are identical, it makes sense to focus on a symmetric equilibrium where  $t_i = t$  and  $k_i = \bar{k}_i = k$ . Furthermore, using (10) and (11), we have the following comparative static results in the symmetric equilibrium.

$$\frac{\partial k_i}{\partial t_i} = \frac{n-1}{n f_{kk}} < 0 \quad (16)$$

$$\frac{\partial k_j}{\partial t_i} = -\frac{1}{n f_{kk}} > 0 \quad (17)$$

$$\frac{\partial \rho}{\partial t_i} = -\frac{1}{n} < 0 \quad (18)$$

Using (16)-(18), (15) will be rewritten as

$$\frac{u_g}{u_x} = \frac{1-m}{1-\epsilon(1-\beta)}, \quad (19)$$

where the subscript ' $i$ ' is omitted again and  $\epsilon$  is the capital demand elasticity with respect to the tax rate evaluated at the symmetric equilibrium,  $\epsilon \equiv -(\partial k_i / \partial t_i)(t_i / k_i) \forall i$ . In the tax competition literature, where there exists no benefit spillovers,  $\epsilon < 1$  is often assumed. However, in this paper,  $\epsilon$  is

allowed to take a value greater than 1, since the assumption we need in this paper is  $\epsilon(1 - \beta) < 1$ .

The optimal provision of public good can be expressed by the same condition as in (8), which can be found by maximizing the sum of utilities with respect to  $g_i$ . From (8) and (19), we find that an equilibrium will be Pareto optimal when

$$m = \frac{\beta(n-1) + \epsilon(1-\beta)}{1 + \beta(n-1)}. \quad (20)$$

For  $\epsilon = 0$ , (20) reduces to (9). From (20), we obtain the following result.

**Proposition 2.** If the elasticity of capital  $\epsilon$  is strictly higher than  $(n-1)/n$ , then the matching grant rate is not a monotonous increasing function of the degree of benefit spillovers.

**Proof.** As an extreme case, compare the matching grant rates in the cases of  $\beta = 0$  and  $\beta = 1$ .  $m(\beta = 1) < m(\beta = 0)$  if<sup>3</sup>

$$\epsilon > \frac{n-1}{n}. \quad (21)$$

The intuition of Proposition 2 is as follows. In the economy described above, there are two sources of inefficiencies; (i) benefit spillovers of public good and (ii) fiscal externality. As often explained in the literature, the cause of fiscal externality lies in that the regional government ignores the external effects of its tax change on the other regions; Although a tax increase in region  $i$  affects the other regions' fiscal budget as  $\sum_{j \neq i} t_j (\partial k_j / \partial t_i)$ , regional government  $i$  does not take it into account [Wildasin (1989)]. However, since

<sup>3</sup>Assuming  $\partial \epsilon / \partial \beta = 0$ , (21) can be obtained by taking derivatives of  $m$  with respect to  $\beta$  since  $\partial m / \partial \beta = [n-1 - \epsilon n + (1-\beta)[1 + \beta(n-1)](\partial \epsilon / \partial \beta) / [1 + \beta(n-1)]^2$ .

there exist benefit spillovers of public goods in the model described above, the regional government takes a part of the external effects into account, and as the degree of spillovers increases the regional government becomes to consider the external effect more. For instance, as shown in the recent article of Bjorvatn and Schjelderup (2002), if  $\beta = 1$  regional government perfectly accounts for the fiscally external effect. Hence,  $\beta$  plays a role in measuring how much fiscal externality the regional governments account for. Though an increase in  $\beta$  increases the spillover externality, it reduces the distortion due to the fiscal externality, so that the optimal matching grant rate might decrease.

The combination of  $(n, \epsilon)$  where (21) holds is shown by the shaded area in Figure 1. For instance, when  $n = 2$ , the rate of the matching grant should be decreased as the degree of spillovers increases if  $\epsilon > 0.5$ . Parry (2003, p.46-47) states that ‘based on the empirical evidence, a range of about 0.1 to 0.8 seems the most plausible’, by referring Bartik (1991) and Brueckner (2001).

For further illustration, consider the familiar Cobb-Douglas production function:  $y_i = k_i^a (0 < a < 1)$ . Now, we define the capital demand elasticity,  $\eta_i \equiv -[\partial k_i / \partial (\rho + t_i)][(\rho + t_i) / k_i] > 0$ , which is given by

$$\eta_i = \frac{1}{1 - a}. \quad (22)$$

In the symmetric equilibrium, using (10) and (22), we have<sup>4</sup>

$$\epsilon_i = \frac{n - 1}{n} \frac{t_i}{(\rho + t_i)(1 - a)} = \frac{n - 1}{n} \frac{t}{a(1 - a)k^{a-1}}, \quad (23)$$

which implies that (21) holds if

$$\frac{t}{a(1 - a)k^{a-1}} > 1. \quad (24)$$

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<sup>4</sup>See Wildasin (1988, 1989).

To provide some insight as to when the matching grant rate should be decreased as the degree of benefit spillover increases, rewrite (24) as

$$\theta > a(1 - a), \quad (25)$$

where  $\theta \equiv tk/k^a$  denotes the share of local public spending in the regional product. Consider the Cobb-Douglas type of utility function,  $u_i = x_i^{1-b}G_i^b$  ( $0 < b < 1$ ). In the symmetric equilibrium, we have  $b = \theta$ , which reveals that (25) is likely to hold to the extent that the resident puts high weight on the public goods. This example is worth noting because it shows that Proposition 2 might hold irrespective of the number of regions.

If we assume the Cobb-Douglas type of functional forms, we have another result.

**Proposition 3.** As the number of regions increases, the optimal matching grant rate increases.

*Proof.* Taking derivatives of  $m$  with respect to  $n$  in (20), we have

$$\frac{\partial m}{\partial n} = \frac{\beta[1 - \epsilon(1 - \beta)] + (1 - \beta)(1 + \beta(n - 1))\frac{\partial \epsilon}{\partial n}}{(1 + \beta(n - 1))^2}.$$

As we assume the Cobb-Douglas functions, we have  $\epsilon = \frac{n-1}{n} \frac{b}{a(1-a)}$ , implying  $\partial \epsilon / \partial n > 0$ . Hence,  $\partial m / \partial n > 0$  since  $1 > \epsilon(1 - \beta)$ .

Proposition 3 is a simple extension of the result of Hoyt (1991) if we consider a reduction in  $n$ . As shown in Hoyt (1991), the decrease in the number of regions reduces the distortion due to the fiscal externality. In addition, in this paper, the reduction in  $n$  reduces the spillover externality.

Two factors act on the direction that reduces optimal grant rate.

## 4 Conclusion

The conventional wisdom is that the optimal size of grant program increases as the degree of benefit spillovers of public goods increases. However, in this paper, we identify another role of spillover that works in the opposite direction. In this paper, we argue that the increase in spillover actually reduces the distortion due to the fiscal externality, so that an optimal matching grant rate might decrease with the increase in the degree of benefit spillovers.

The benefit spillover of public goods is an inevitable phenomenon both in the contexts of regional governments within a country and of national governments within the world economy. Our finding is significant in the policy aspect since it tells that we do not have to have an extensive subsidy program, implemented by supra-regional government, when we face the increase in the benefit spillover of public policies.

Finally, it should be noted that some of the assumptions can be relaxed without changing the main result (Proposition 2) of this paper. Specifically, the symmetry assumption allows us to derive clear cut result but some relaxation of this assumption would only weaken the result in a quantitative sense; Allowing asymmetric regions only changes the square of the shaded area in Figure 1.

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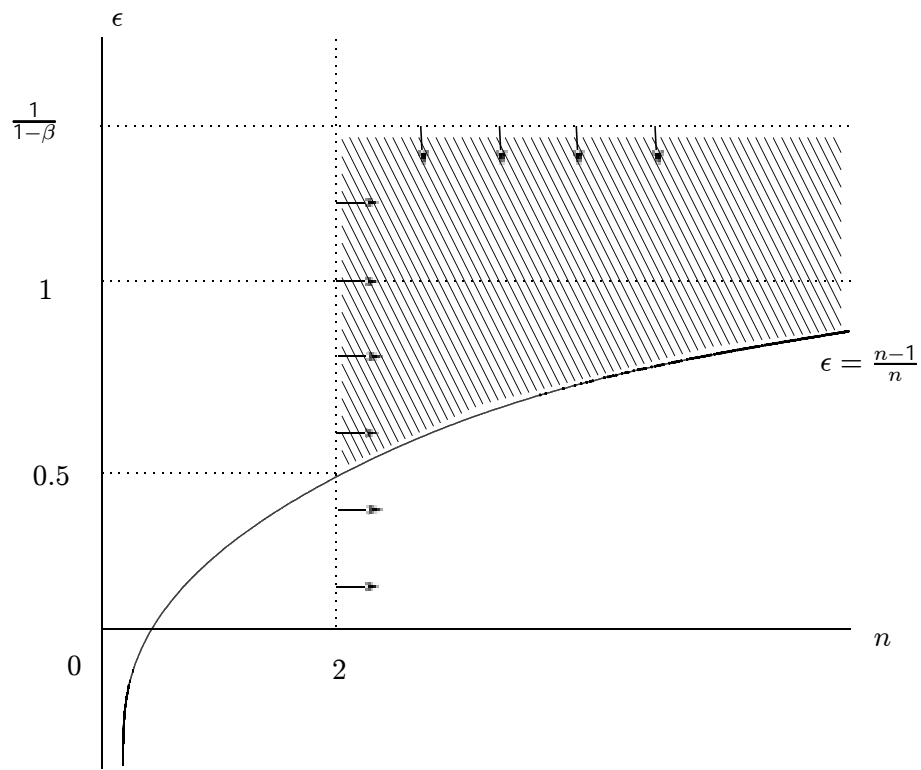


Figure 1. The area where  $m(\beta = 1) < m(\beta = 0)$  holds.

Note:  $0 < \beta < 1$ ,  $n \geq 2$  and  $\epsilon < 1/(1 - \beta)$ .