ECONOMIC RESEARCH CENTER
DISCUSSION PAPER

*E-Series*

No.E14-13

# Entrepreneurship, Financial Intermediation, and Inequality

by

Takanori Adachi

December 2014

ECONOMIC RESEARCH CENTER
GRADUATE SCHOOL OF ECONOMICS
NAGOYA UNIVERSITY

# Entrepreneurship, Financial Intermediation, and Inequality*

Takanori Adachi†

November 19, 2014

**Abstract**

This paper provides a simple dynamic framework to study the long-run relationship between financial intermediation and wealth inequality. By considering two types of entrepreneurial financing (self-financing and intermediated financing), I show that wealth inequality is more severe in an economy where all financing is intermediated than in an economy where some entrepreners rely on self-financing. This result is consistent with the augmented Kuznets hypothesis that a large scale operation of production and the financial intermediary development are associated with higher inequality.

Keywords: Entrepreneurship; Financial Intermediation; Inequality.

JEL classification: E21, L26.

*I am grateful to Munetomo Ando, Ryo Arawatari, and Makoto Nakajima for helpful discussions on an early version of the paper. I also acknowledge a fiancial grant from the the Yamada Foundation and a Grant-in-Aid for Scientific Research (A) (23243049) from the Japanese Ministry of Education, Science, Sports, and Culture. Any remaining errors are my own.

†School of Economics, Nagoya University, 1 Furo-cho, Chikusa-ku, Nagoya 464-8601, Japan. E-mail: adachi.t@soec.nagoya-u.ac.jp

# 1 Introduction

What is the relationship between financial intermediation and inequality? Consider the case where one needs to raise outside capital for production. Under capital market imperfections, one's borrowing opportunities would be associated with his or her wealth level. On one hand, financial intermediation might ease inequality by making less wealthy people capable to borrow. On the other hand, one may also expect that financial intermediation would help richer people borrow more and more, accelerating inequality. The same arguments also hold for the situation where opportunities of financial access become broadened by financial liberalization such as deregulation or capital account liberalization.

Although it is difficult to tell whether inequality *per se* is beneficial or harmful, it would be important to understand how the degree of inequality in a society is determined (see, e.g., Piketty (2014)). One may argue that inequality matters to the working of social infrastructure such as political stability (see, e.g., Alesina and Perotti (1996)) and public safety (see, e.g., Fajnzylber, Lederman, and Loayza (1998)). If so, policymakers would be concerned about what determines the degree of inequality. This might be the case especially when they want to consider the balance of economic organization and non-economic arrangement in their society. Traditionally, what they are expected to do to cure inequality is to engage in redistribution, that is, *ex-post public* transfers. However, they may also want to know *ex-ante private* factors which generate inequality, given that they have an appropriate incentive and instrument to affect inequality.

Taking inequality per se seriously, this paper provides a simple dynamic framework to study the long-run relationship between financial intermediation and wealth inequality. More specifically, I consider a deterministic dynamic model à la Matsuyama (2000), who analyzes how inequality arises in an economy with an imperfect capital market. In contrast to Matsuyama (2000), my model allows different types of finance (and can eliminate the steady state equilibrium with perfect equality). I assume that some positive fixed amount of capital is necessary to do production (i.e. non-convex technology). Then, in the presence of the capital market imperfection (due to imperfect enforcement), a household with little wealth cannot borrow anywhere even if he wants to start production, while he may borrow from a financial intermediary when he has enough wealth. A borrower always has an option of simply defaulting, and as a result, repayment is enforceable only with some inevitable cost. The role of financial intermediaries (FIs) is thus to alleviate this enforcement problem by doing some costly monitoring activities. Specifically, the model below assumes that when the borrower defaults (which never happens in equilibrium, though),[1] FIs can get back some fraction of money (one can think of FIs in the model as local banks, investment banks, securities agencies, and so on). Intermediated borrowing, however, might not be so beneficial for entrepreneurs with a larger amount of wealth because it is costly in nature. This paper aims to capture this feature in a formal model. While uncertainty nor asymmetric information is not considered to maintain analytical tractability, I provide a full characterization of the steady-state equilibria to obtain interesting insights about the relationship between finance and inequality.

Specifically, I provide a theoretical basis for the augmented Kuznets hypothesis (see the next section for the related literature); (i) *inequality persists in a country and varies across countries*, and (ii) *a large scale operation of production and the financial intermediary development are associated with higher inequality.* I consider two types of financing, self-financing and intermediated financing, and assume that there is imperfect enforcement in the capital market, which makes room for financial intermediaries endowed with the monitoring technology to play a role in the economy. Other economic agents are households (potential entrepreneurs), who are heterogenous only in the level of their wealth. In the analysis below, the interest rate is endogenously determined. I provide the steady-state characterization to see what kind of financial pattern prevails in the economy as well as the characteristics of the wealth distribution. Although there is no such heterogeneity as talent, perfect equality never arises in any steady-state equilibria (i.e., inequality persists) unless the initial wealth distribution is too skewed toward rich or poor. It is also found

[1] Note also that default is always voluntary; since there is no uncertainty in the present paper, debt overhang is not an issue.

that wealth inequality is severe for a lower equilibrium interest rate, and that wealth inequality is more severe in an economy where all financing is intermediated than in an economy where some entrepreners rely on self-financing. It is also shown that for a wide range of parameters (concerning the benefit and the cost of monitoring) there are two continua of the steady-state equilibria; one is where all entrepreneurs rely on financial intermediation, and the other is where some of (richer) borrowing entrepreneurs self-finance. The multiplicity comes from the mutually reinforcing effects; in the former type of equilibria, the equilibrium interest rate is low and the supply of capital (the number of poorer agents) is large. These two effects are mutually dependent, and inequality is severe. In the second type of equilibria, however, the equilibrium interest rate is high and the supply of capital (the number of poorer agents) is small. Inequality is less severe. These features are consistent with the findings of Clarke, Xu, and Zou (2013) (see the next section). My formal model suggests that even if two economies that have similar values of parameters, they may end up different types of economy in terms of financial structure.

The rest of the paper is organized as follows. The next section briefly reviews the related literature. Then, I present a dynamic model in Section 3. Section 4 provides the steady-state analysis, followed by Section 5 where the effects of capital account liberalization are discussed. Section 6 concludes the paper.

## 2 Related Literature

This paper is related to the literature which investigates macroeconomic or development implications in the presence of capital market imperfections, which is started with a seminal article by Galor and Zeira (1993).[2] These papers are concerned with the long-run effects of the imperfect capital market on the wealth distribution. They are basically silent on the *differences in finance*. An exception is a paper by Chakraborty and Ray (2006), who investigate the issue of bank-based versus market-based lending in an $Ak$-type endogenous growth model. Specifically, they

[2]Followed by, e.g., Banerjee and Newman (1993), Aghion and Bolton (1997), Piketty (1997), Matsuyama (2000) and many others.

extend Holmström and Tirole's (1997) incentive model (moral hazard with respect to the project choice) of financial intermediation to a dynamic context. As in the present paper, the role of bank monitoring is to mitigate the agency problem, and each entrepreneur chooses how she borrows the working capital. Chakraborty and Ray (2006) focus on the balanced growth paths, and compare the growth rate of per capita GDP (and other macro variables) in the market-based system with the one in the bank-based system. In their model, however, there is no room for a mixed structure of different finance types to arise: in the long run, all entrepreneurs (except for the ones who can borrow from nowhere) in the economy borrow either from banks or from markets, depending on exogenous parameter values concerning monitoring effects and costs. In other words, direct and intermediated lending *cannot coexist* in an economy in Chakraborty and Ray's (2006) model. On the contrary, Chakraborty and Ray (2007) allow three types of households to emerge in the steady state; (i) those who cannot borrow, (ii) those who borrow some amount from a bank, and (iii) those who rely only on the credit market. They focus on two features of a financial system; its depth and structure. By financial depth, they mean how large the proportion of unconstrained borrowers is. Financial structure captures the fraction of borrowers who rely only on the market among them. Basically, the initial inequality entirely determines to which financial system the economy converges; the more unequal in the initial stage, the less developed the economy's financial system remains. This is because Chakraborty and Ray (2006, 2007) consider a small open economy. Namely, they assume that the interest rate is exogenously given. In the following model, the interest rate is endogenously determined as a component of equilibrium. As such, the initial distribution is just one factor to determine the characteristics of the steady state.

The formal model presented below is also motivated by empirical findings on inequality.[3] Based on a time series data of England, Germany and the US available

[3]Note that the target of these empirical studies is on *income* inequality rather than *wealth* inequality. This is because it is not easy to find an appropriate index or a good proxy for wealth inequality (the GINI index for land ownership is typically used). Although it is well held that wealth inequality is more severe than income inequality, we do not go into details on this issue in the present paper. The benefit of considering a theoretical model is that one can systematically

at that time, Kuznets (1955) attempted to offer a broad hypothetical view on the relationship between economic development and income inequality, which has been known as the *Kuznets inverted U-shaped hypothesis*. It states that as an economy develops income inequality rises, but in the later stage of development it mitigates. Kuznets (1955) attributed this change to the migration shift from the traditional agricultural sector to the modern industrial sector, where the wage dispersion is large. The congestion in the modern sector eventually makes the traditional sector again attractive, which eases inequality.

Different from Kuznets' (1955) original reasoning, Greenwood and Jovanovic (1990) offer a theoretical model to endogenously derive the inverted U-shaped curve, focusing on the role of *financial intermediation*. In their model, individuals can invest in a risky but profitable project only when they pay a fixed membership fee to join a financial intermediary coalition. This fixed cost first has a role to prevent poorer individuals from accumulating wealth, which exacerbates inequality, However, the more rich individuals join these coalitions, the lower the entry fee becomes (because the average cost of the coalition declines as the number of members increases), which eventually vanishes inequality.

However, a number of empirical studies (such as Deininger and Squire (1998), Li, Squire, and Zou (1998), and Clarke, Xu, and Zou (2013)) find little support for the Kuznets inverted U-shaped relationship between income inequality and the level of income per capita. Rather, as Li, Squire, and Zou (1998) show, the degree of inequality seems to have a nature to *persist* within an economy, but it varies across economies. Based on their cross-country empirical study, Clarke, Xu, and Zou (2013) also cast doubt on the role of financial intermediation in Greenwood and Jovanovic's (1990) story. Considering these points, I model inequality as a perpetuating phenomenon even in the long run. In particular, equality never arises in any steady state equilibria in my model. This is in sharp contrast to the models which derive wealth distribution but allow perfect equality to arise as one of the equilibria.[4] I also interpret institutional differences in finance as main causes of

deal with both inequalities.

[4]Mookherjee and Ray (2003) is an exception.

generating these international varieties in equality, if other possibly related factors are controlled.

Even though Kuznets' (1955) original inverted U-shaped hypothesis has gained little empirical support, Kuznets' (1955) analysis could be still insightful. If the modern technology which entrepreneurs adopt needs high leverage, financial intermediation would help already rich people borrow more, keeping poorer households from starting the project, who remain suppliers of capital. In this way, inequality might be associated with the prevalence of the modern technology via financial intermediation. Indeed, Clarke, Xu, and Zou (2013) find this effect in their empirical study, and call it an *augmented Kuznets hypothesis*: as Kuznets (1955) suggested, sectorial structure matters to inequality. In particular, a large scale operation of production (measured by added value of non-agricultural sectors divided by GDP) and the financial intermediary development (measured by the amount of bank assets or of private credit divided by GDP) are associated with higher inequality (measured by the GINI index of income). To the best of my knowledge, there is no theoretical model that formalizes this effect of financial intermediation on inequality. Although my simple model cannot replicate all of their empirical results, one may find it useful in terms of investigating the role of financial intermediation from various points of view.

It is also interesting to see the effects of capital account liberalization on inequality. Based on the data of eleven emerging markets which ex equity market liberalization during the period of 1986 to 1995, Das and Mohapatra (2003) find that average middle class income share (*not* the absolute value) decreased while the average income share of the highest class increased (and the lowest class experienced little change in their share). After analyzing the steady-state, Section 6 incorporates capital account liberalization into the dynamics of my model, and considers its effects on the wealth distribution.

# 3 The Model

In this section, I describe a formal dynamic model of household behavior and financial intermediation. In particular, I incorporate financial intermediation into a dynamic model à la Matsuyama (2000). I first explain the production technology, imperfect enforceability, and the role of financial intermediation. Then, I determine the equilibrium interest rate in each period, and illustrate equilibrium dynamics of the wealth distribution and the interest rate.

## 3.1 Economic Environment

The economy is closed with an infinite, discrete time horizon $t = 0,1,2,....$ The word "closed" implies that the interest rate is *endogenously* determined in the model. In Section 5, I consider the effects of capital account liberalization on the dynamics. In this economy, there is a continuum of dynastic families which live forever. The total mass is normalized to be one, and there is no population growth. Each agent in a dynastic family is risk-neutral and lives for one period only (reproducing one son).[5] Also, there is a competitive financial sector which consists of a large number financial intermediaries (FIs). I assume that intermediaries and households are different agents.

In this economy, there is only one type of good, which can be consumed or be made for bequest (to be explained shortly). In each period $t$, an identical household has the following deterministic production technology,[6] which is non-convex (because of discontinuity):

$$f(k_t) = \begin{cases} k_t & \text{if } k_t \geq q, \\ 0 & \text{if } 0 \leq k_t < q, \end{cases}$$

where $k_t \geq 0$ is his investment level and the unit revenue is normalized one,[7] which

[5] The male pronouns are used throughout to represent a generic agent.

[6] Under the fixed cost of starting production and borrowing constraints (like the ones introduced later), households might consider joint borrowing and joint production. I simply assume away this possibility in the present paper.

[7] In the following analysis, this normalization will make the equilibrium per-unit interest rate always less than one, which seems odd at first (because repayment is less than the amount of

accrues when his investment is over the normalized fixed level, which is not too large but not too small, either ($k_t = q \in [1, 2)$). This fixed cost can be literally taken as physical capital, or can be thought of entrepreneurial human capital. Note that the output is linearly increasing after the fixed threshold level of capital. I assume that each household has access to an alternative "backyard" storage technology, whose per-unit return is $\rho \geq 0$ for any input level (i.e. no fixed cost is necessary to generate a return). This can be also considered as a traditional technology such as small-scale agriculture. Restrictions on $\rho$ will be added later on. We also assume that each household earns an exogenous nonrandom revenue, which is normalized to one and is common to all households and non pecuniary (so households cannot borrow or lend a part of this income). This is just a technical assumption to yield steady-state results in this non-growth model.[8]

Let $a_t \geq 0$ denote the wealth of a household in generation $t$ (which is inherited from his parent at $t-1$; to be explained in Subsection 3.6). The wealth level is the only source of household heterogeneity. The distribution of wealth across households is denoted by the measure $G_t(a)$ defined on Borel subsets of $[0, \infty)$, and the initial wealth distribution $G_0(a)$ is given.

Given his inherited wealth $a_t$ (and under the constraints explained later), a household maximizes his income. He can consume it by himself or can make it bequest to his son at $t+1$ (to be explained in Subsection 3.6). We call a household an *entrepreneur* when he earns revenue from using the production technology. For simplicity, we assume that capital for production fully depreciates in one period. This assumption would particularly be fitted when the capital is interpreted as human capital. In this economy there is another revenue-generating opportunity for households (other than the backyard storage technology), namely a one-period competitive capital market (to be explained in the next paragraph), where a per-unit gross interest rate $r_t \geq 0$ accrues when a household has saved some of his wealth in

borrowing!). However, for analytical purposes, it is more important to note that it still earns a positive amount, and this normlization does not invalidate the main thrust of the results.

[8]This exogenous income should not be large; otherwise, all the households become rich enough and they can easily overcome the investment threshold caused by the non-convexity of the technology. It can be verified that normalizing it to one satisfies this requirement.

an FI. Thus, the opportunity cost of using capital $k_t$ for production is $\max\{\rho, r_t\}k_t$.

The timing of decision making in period $t$ is as follows. At the beginning a new agent in a household is given the inherited wealth from his parent. He divides it into savings on the backyard storage ($s_t$) and the rest ($a_t - s_t$). Then, he decides whether he becomes an entrepreneur or not, and he divides ($a_t - s_t$) into the part for production (only when he becomes an entrepreneur) and the part for savings in the savings market.

## 3.2 Financial Arrangement

Although as a deposit saver a household can save any amount of money in an FI, he cannot borrow any amount from an FI. In addition, he cannot borrow directly from other households; he must borrow from an FI, otherwise, he must self-finance. These features come from the enforcement problem and the role of FIs, which is explained below in this subsection. First, we look at the role of FIs adopted in the present paper.

Following Holmström and Tirole (1997), I assume that the role of FIs is to *monitor* an entrepreneur to mitigate his opportunistic behavior. I also assume that only FIs are endowed with monitoring technology, or one may assume that households have a prohibitively costly monitoring activity because of, say, the lack of specialization. Monitoring is a broad concept, and one can think of various types of monitoring. In my model, monitoring takes place when lending is made, and it determines what the lender can do in the case of default. As such, it may also include the cost of writing a contract regarding what will be legally done in the case of default, and/or the cost of FIs' service (via renegotiation) in the case of default. It may include sending experts to the company boards. FIs offer a fixed interest rate $r_t$ on savings deposits, and earn revenue using the deposits to make loans to entrepreneurs who borrow from them. The rate charged for a loan made by an FI is the (gross) lending rate $i_t$. The lending-deposit rate spread is the return to the FI for providing the financial service. An FI will choose whether to monitor or not; there is no choice for intensity of monitoring. For simplicity, I assume that there is no fixed cost for monitoring and that the marginal cost of monitoring (per the amount of lending) is $\gamma > 0$, which is exogenous[9] and constant over time (that is, the monitoring technology has constant returns to scale). This can be understood as FI's disutility of labor for monitoring, or their human capital value (both of which are assumed not to be tradeable). I also assume that the monitoring cost is sunk when lending is made (in this sense monitoring here is *ex-ante* one so that free riding is not an issue). In addition, I assume that there is a large number of FIs. Perfect competition in the financial sector implies that deposits each FI receives are equal to loans issued by that FI and

$$i_t = r_t + \gamma \tag{1}$$

so that FIs make zero profit in equilibrium.[10] Note that the possibility of FI's incentive problem is assumed away. One may think that an FI can make a credible commitment, caring about its reputation.

If there were no enforcement problem, however, no household wants to borrow from an FI because it should be more expensive than when he can borrow directly from other households.[11] In order to validate the existence of FIs, I assume that *enforcement in the capital market is imperfect*; an entrepreneur always has an option of simply defaulting.[12] Specifically, suppose that he borrows $b_t$. When the borrower does not honor his repayment $i_t b_t$, however, the lender cannot seize all of the entrepreneur's revenue due to the imperfect enforcement. In other words, an entrepreneur can pledge only up to some fraction of his revenue. This amount is called his *pledgeable* revenue, and the following modeling assumption is made.

**Assumption 1.** *Ex-ante monitoring by an FI is necessary for the borrower's*

[9] Ando and Yanagawa (2004) construct a model where monitoring technology is endogenous.

[10] We do not assume monotoring has increasing returns to scale, which is incompatible with perfect competition. See Allen and Gale (2000, Ch.8) and Allen and Gale (2004) for an analysis of competition in the banking sector.

[11] And, if the interest rate is below one, he wants to borrow an infinite amount of capital because of the linearity of the production technology. As is seen below, the equilibrium interest rate under the perfect enforcement should be one.

[12] In the present paper, I simply assume away the role of intertemporal incentives such as reputational concerns. Since my interests lie in macroeconomic issues, and I do not pose microeconomic problems in, say, a small community, this would not cause a serious flaw.

*pledgeable revenue to be positive.*

Specifically, I assume that when an entrepreneur tries to circumvent intermediated borrowing, his pledgeable revenue is zero.[13] It becomes $\lambda k_t$, however, when he relies on intermediated borrowing. Then, Assumption 1 is expressed by $0 \leq \lambda < 1$.[14] Note that $\lambda$ is less than one, meaning that FIs can improve but cannot perfectly correct the enforcement problem. The parameter $\lambda$ can be understood as capturing the effectiveness of monitoring. It may be strengthened by the efficiency of the economy's legal system for protecting investors (but the financial sector is necessary to work it).[15] This can be interpreted as the situation where upon default the fraction $1 - \lambda$ of the production revenue perishes due to the costly renegotiation process. One may also interpret this as a reminiscent of the costly state verification (CSV) problem; verifiability of the project is not without cost.[16]

From the borrower's side, these pledgeable revenues are also the default cost, which is seized by the lender upon default.[17] Thus, if an entrepreneur circumvents intermediated borrowing, he cannot borrow $k_t$ and save $s_t$ in the backyard storage technology over his wealth level $a_t$. That is, he can borrow money within his wealth level which he has saved. In this sense, we call this situation *self-financing*. This is

[13] This might seem a strong assumption. However, if one allows non-zero pledgeable income when an entrepreneur escapes intermediated borrowing, the steady state analysis becomes complicated, and yields less interesting results.

[14] Note that $\lambda = 1$ means the perfect enforcement: as is shown below, any household with $w_t \geq 0$ does not suffer from the borrowing constraint (to be formalized shortly).

[15] Seminal papers on the international differences of the legal system in protecting investors are La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1997, 1998).

[16] For more on the formulation of this problem, see, e.g., Freixas and Rochet (2008).

[17] Note that Kiyotaki and Moore's (1997) justification for the borrowing constraint can also be applied in the present context. They propose the following story. Production technology is specific to the borrower, and if the lender succeeds the borrower's production he cannot do production as much as the lender can do. Knowing this, the lender can suggest renegotiating the initial contract to reduce his repayment, and the borrower will accept this as long as the suggesed repayment is not below the level of he can earn by succeeding the borrower's production. If one assumes that it is the lender who must succeed the borrower's production, and that (unmodeled) "shadow" middilemen, who have an inferior technology, are actually between lending households and borrowing entrepreneurs, then Kiyotaki and Moore's (1997) story applies. If the relationship between lending households are borrowing entrepreneurs is literallhy direct, then it does not hold. However, this seems less natural.

expressed by

$$r_t(k_t - a_t + s_t) \leq 0. \tag{2}$$

We assume that households are allowed either to self-finance or to borrow all his capital from an FI only. Then, the borrowing constraint under intermediated borrowing becomes

$$(r_t + \gamma)(k_t - a_t + s_t) \leq \lambda k_t. \tag{3}$$

Note that in equilibrium default *never* happens since this inequality must hold.[18] I also assume limited liability; borrower's payment cannot exceed his total income.

## 3.3 Optimal Investment Decisions

Now we turn to the revenue structure of households. Consider a household with $a_t$ and suppose that he saves $s_t \leq a_t$ in the backyard storage technology (this does not happen in equilibrium in the analysis below). If he invests $k_t$, and when he self-finances (that is, $k_t \leq a_t - s_t$), his total income becomes $1 + \rho s_t + F(k_t) + r_t(a_t - s_t) - \max\{\rho, r_t\}k_t$. So, it can be written as

$$\begin{cases} 1 - (r_t - \rho)s_t + \min\{1 - \rho, 1 - r_t\}k_t + r_t a_t & \text{if } k_t \geq q, \\ 1 - (r_t - \rho)s_t + r_t a_t - \max\{\rho, r_t\}k_t & \text{if } 0 \leq k_t < q. \end{cases}$$

On the other hand, when he borrows from an FI, it becomes $1 + \rho s_t + F(k_t) + r_t(a_t - s_t) - \max\{\rho, i_t\}k_t$ (with $k_t$ satisfying constraint (3)). Thus, it can be written as

$$\begin{cases} 1 - (r_t - \rho)s_t + \min\{1 - \rho, 1 - i_t\}k_t + r_t a_t & \text{if } k_t \geq q, \\ 1 - (r_t - \rho)s_t + r_t a_t - \max\{\rho, i_t\}k_t & \text{if } 0 \leq k_t < q. \end{cases}$$

Here, it is implicitly assumed that a household saves $a_t - s_t$ on deposits in an FI and after that he draws for self-financing or borrows from an FI. It is seen that given his investment level $k_t$ only households whose wealth satisfy $a_t - s_t \geq k_t$ can

[18] A similar borrowing constraint is adopted by Holmström and Tirole (1997), where production is stochastic. This makes dynamic analysis less tractable. Indeed, Chakraborty and Ray (2006) incorporate Holmström and Tirole's (1997) incentive problem into the dynamic model. However, they assume that the most efficient technology out of three is deterministic.

self-finance his investment to become an entrepreneur. Similarly, a household whose wealth after the saving in the storage technology $a_t - s_t$ is greater than or equal to $[1 - \lambda/(r_t + \gamma)]k_t$ can borrow capital from an FI. Since the minimum investment level to become an entrepreneur is $k_t = q$, only a household with $a_t - s_t \geq q$ can become an entrepreneur by self-financing. Similarly, a household whose net wealth $a_t - s_t$ is greater than or equal to $[1 - \lambda/(r_t + \gamma)]q$ can borrow capital from an FI.

Now, in order to analyze the effects of the enforcement problem, I make the following assumption.

**Assumption 2.** $\lambda > \gamma + \rho$.

This assumption states that the monitoring cost is not so high, and hence monitoring is socially desirable given the enforcement problem. It can be also stated that the monitoring effect is high enough, and that the backyard storage technology is not so productive. Note that this assumption implies $\beta\rho < 1$.

Notice at this moment that if there were no enforcement problem then no households would be free from borrowing constraint (and there would be no financial sector). As a result, the equilibrium interest rate would be $r_t = 1$, which is the only possible case. This is because if $r_t > 1$, all households, irrespective of his wealth $a_t$, would want to become a lender, and if $r_t < 1$, all households would want to become an entrepreneur, both of which imply the capital market would not clear. Note that no households want to use the backyard storage technology because $\rho < 1$. In this case, all households are indifferent between borrowing and lending, obtaining $1 + a_t$. So, how the wealth in the economy is divided to lending and borrowing is indeterminate. The GDP in this economy is one (the identical revenue times the population) for any period.[19] These arguments can be summarized in the following proposition.

**Proposition 1.** *There is no income inequality across households if there are no*

[19]In the following analysis, I ignore the value of the exogenous revenue when we calcurate the aggregate income measures such as the GDP and the GINI index. This eases computation, and enables more direct intermpretation of the results.

*enforcement problems (the GINI index for earnings inequality is zero). That is, irrespective of the wealth level, all households earn a net inflow one in any period.*

Note that there can be *income* inequality in period $t$: given the wealth level $a_t$, a household obtains $1 + a_t$. However, as we will see in the next section, this income inequality and wealth inequality in the perfect world disappears in the long run. This is also a consequence of the assumption that there is no talent or endowment heterogeneity.

On the contrary, if there were no financial intermediary sector in our imperfect world, income inequality would arise. It is easy to see that a household has no way to use the backyard storage technology if his wealth level $a_t$ is less than $q$, earning per-unit income $\rho$ (his total revenue is $1 + \rho a_t$), while it will earn one if he has $a_t \geq q$ (his total revenue is $1 + a_t$). The aggregate earnings in period $t$ in this case is $\rho G_t(q) + (1 - G_t(q)) = 1 - (1 - \rho)G_t(q)$. The GINI index for income inequality is calculated as

$$\begin{aligned} IncomeGINI_t^{NFI} &= 1 - \frac{\rho[G_t(q)]^2}{1-(1-\rho)G_t(q)} - \left(\frac{\rho G_t(q)}{1-(1-\rho)G_t(q)} + 1\right)[1 - G_t(q)] \\ &= \frac{(1-\rho)G_t(q)(1-G_t(q))}{1-(1-\rho)G_t(q)}, \end{aligned}$$

from which we have $\partial IncomeGINI_t^{NFI}/\partial\rho < 0$; the more efficient the storage technology is the less severe income inequality is. The sign of $\partial IncomeGINI_t^{NFI}/\partial G_t(q)$ is ambiguous. This is because an increase in $G_t(q)$ can imply either upward or downward shift. In the next section, wealth inequality in this case will be considered.

Now, I come back to the world with the imperfect enforcement and financial intermediation to investigate the effects of financial intermediation on equality. First, I obtain the following lemma about the lower bound of the equilibrium interest rate.

**Lemma 1.** *In equilibrium, it must be the case that $r_t > \lambda - \gamma$.*

**Proof.** Suppose that $r_t \leq \lambda - \gamma$ happens. Then, the borrowing constraint (3) is no longer binding for any household so that every household with $a_t - s_t < [1 - \lambda/r_t]q$

demands infinite capital, meaning there is excess demand of capital in this economy. *QED*

Using this lemma, I obtain the following lemma that states that savings in the backyard storage technology do not happen in equilibrium.

**Lemma 2.** *In equilibrium, no households use the storage technology. That is, for any household,* $s_t = 0$ *for any* $t$*.*

**Proof.** Suppose $s_t > 0$ for some $t$. Then, by reducing some amount of $s_t$ and by placing that on deposits in an FI, a household obtains, by Lemma 1, a per-unit gain $r_t > \lambda - \gamma$, which is greater than the backyard storage technology's return $\rho$ by Assumption 2. *QED*

Since a household does not choose $s_t = 0$, it is seen that given his investment level $k_t$ only a household with $a_t \geq q$ can become an entrepreneur by self-financing (as is explained below, he may optimally borrow from an FI). Similarly, a household whose wealth $a_t$ is greater than or equal to $[1 - \lambda/(r_t + \gamma)]q$ ($\equiv \underline{a}(r_t)$) can borrow capital from an FI (as is also explained below, he may optimally self-finances when his wealth is high enough). Notice that $\underline{a}(r_t) < q$ for any $r_t > \lambda - \gamma$.

Since $r_t$ is endogenously determined, the threshold $\underline{a}(r_t)$ is also endogenously determined as a function of $r_t$, while $q$ is not. Simple algebra shows that:

$$\frac{\partial \underline{a}(r_t)}{\partial r_t} = \frac{\lambda q}{(r_t + \gamma)^2} > 0, \; \frac{\partial^2 \underline{a}(r_t)}{\partial r_t^2} = \frac{-2\lambda q}{(r_t + \gamma)^3} < 0,$$

which means that $\underline{a}(\cdot)$ is strictly increasing and concave. This fact will be used in the next section. The first inequality shows that the higher the interest rate the tighter borrowing constraint (3) is. The latter relationship implies that $\underline{a}(r_t)$ is concave with respect to $r_t$. Then, the following lemma is obtained.

**Lemma 3.** *For all* $r_t > \lambda - \gamma$*,* $\underline{a}(r_t) > 0$*. That is, if* $G_t(\underline{a}(r_t)) > 0$*, there exist households (with* $a_t < \underline{a}(r_t)$*) who cannot obtain any external finance so that they become a net lender.*

**Proof.** It is immediate from $\lim_{r_t \downarrow \lambda - \gamma} \underline{a}(r_t) = 0$ and $\partial \underline{a}(r_t)/\partial r_t > 0$. *QED*

It is also verified that:

$$\frac{\partial \underline{a}(r_t)}{\partial q} = 1 - \frac{\lambda}{r_t + \gamma} > 0, \frac{\partial \underline{a}(r_t)}{\partial \lambda} = \frac{-q}{r_t + \gamma} < 0, \frac{\partial \underline{a}(r_t)}{\partial \lambda} = \frac{\lambda q}{(r_t + \gamma)} > 0,$$

which means that the more the necessary amount of the fixed capital, the less effective the monitoring, and the more costly the monitoring, the severer the threshold $\underline{a}(r_t)$ is.

If the interest rate is exogenous from the initial period, then $\underline{a}(r_t)$ is always a constant. Thus, the initial wealth distribution $G_0$ completely determines who can be a borrower and who remains a lender, and there is no social mobility across entrepreneurs and non entrepreneurs because there is no such jumping process as uncertainty in the present model. If the interest rate is endogenous, however, the initial wealth distribution is not a sole determinant. Also, if capital account liberalization occurs in some period $t$ (which means that all agents in this economy take the interest rate as exogenously given), the threshold becomes a constant from that period on. Conversely, suppose that the government stops regulation on the interest rate in some period $t$. Before that period the interest rate is exogenous on the interest rate, but now it becomes endogenous.

The following lemma determines the upper bound of the equilibrium interest rate.

**Lemma 4.** *In equilibrium, it must be the case that* $r_t \leq 1 - \gamma$.

**Proof.** Suppose that $r_t > 1 - \gamma$ holds. Then, a household with $a_t < q$ does not become an entrepreneur. This is because he can earn $1 + r_t a_t$ by choosing $k_t = 0$, while his income is $1 + (1 - \gamma - r_t)k_t + r_t a_t$, which is less than $1 + r_t a_t$, when he becomes an entrepreneur. Thus, intermediated borrowing is relatively costly compared to the benefit from production, so there are no households who borrow from an FI, meaning there is excess supply of capital in this economy. *QED*

I then obtain the following lemma about the optimal amount of capital, given the financial decision (self-finance or intermediated finance).

**Lemma 5.** *In equilibrium, an entrepreneurs chooses* $k_t = a_t$ *when he self-finances, and* $k_t = \overline{k}(a_t, r_t) \equiv a_t/(1 - \lambda/[r_t + \gamma]) > a_t$ *when he borrows from an FI.*

**Proof.** If $r_t < 1 - \gamma$ is the case, then his income is strictly increasing in $k_t$ irrespective of whether he self-finances or borrows from an FI. Thus, he wants to borrow up to the level where the borrowing constraint (2) or (3) binds. If $r_t = 1 - \gamma$, then households with $a_t \in [\underline{a}(1 - \gamma), q)$ are indifferent between becoming an entrepreneur and choosing $k_t = 0$. *QED*

Note here that if the optimal investment level is allowed to be less than his wealth, he may not want to save his money if he is qualified to borrow only from a bank. My formulation on the production technology excludes this situation.

## 3.4 Optimal Financial Decisions and Income Inequality

First, it is immediate to see that when an entrepreneur has no access to capital his income is strictly smaller than the one he can borrow from an FI. This is because the total income when he self-finances is $1 + a_t$, while the one when he borrows from an FI is

$$1 + \frac{1 - \lambda r_t/(r_t + \gamma) - \gamma}{1 - \lambda/(r_t + \gamma)} a_t,$$

and for $r_t < 1 - \gamma$, we have $r_t < [1 - \lambda r_t/(r_t + \gamma) - \gamma]/[1 - \lambda/(r_t + \gamma)]$. Note here that an entrepreneur who can self-finance earns the same amount of income he could earn in the perfect world.

Now, it can be verified that $[1 - \lambda r_t/(r_t + \gamma)]/[1 - \lambda/(r_t + \gamma)] > r_t \Leftrightarrow r_t < 1 - \gamma$, which is assured by Assumption 2. So, both incomes are strictly increasing functions of $a_t$. By considering the two slopes 1 and $\{[1 - \lambda r_t/(r_t + \gamma)] - \gamma\}/[1 - \lambda/(r_t + \gamma)]$, one obtains the following lemma.

**Lemma 6.** *For any* $\lambda \in [0, 1)$ *and any* $\gamma \in (0, \lambda - \rho)$ *there exists a unique* $\widetilde{r} \in (\lambda - \gamma, 1 - \gamma)$ *such that for* $r_t \in (\lambda - \gamma, \widetilde{r}]$

$$[1 - \lambda r_t/(r_t + \gamma) - \gamma]/[1 - \lambda/(r_t + \gamma)] \geq 1$$

*holds, where the equality holds if and only if* $r_t = \widetilde{r}$*. For* $r_t \in (\widetilde{r}, 1-\gamma)$*, it is that*

$$[1 - \lambda r_t/(r_t+\gamma) - \gamma]/[1 - \lambda/(r_t+\gamma)] < 1.$$

**Proof.** The slope of the entrepreneur's income when he borrows from an FI is continuous, differentiable and strictly decreasing in $r_t \in (\lambda-\gamma, 1-\gamma)$ because

$$\begin{aligned}
&\frac{\partial}{\partial r_t}\left(\frac{1 - \lambda r_t/(r_t+\gamma) - \gamma}{1 - \lambda/(r_t+\gamma)}\right) \\
=\;& -\frac{\lambda\gamma}{(r_t+\gamma)^2}\left(1 - \frac{\lambda}{r_t+\gamma}\right) - \frac{\lambda}{(r_t+\gamma)^2}\left(1 - \frac{\lambda r_t}{r_t+\gamma} - \gamma\right) < 0.
\end{aligned}$$

Now, it is verified that

$$1 > 1-\gamma = \lim_{r_t \uparrow 1-\gamma} \frac{1 - \lambda r_t/(r_t+\gamma) - \gamma}{1 - \lambda/(r_t+\gamma)}$$

and

$$\lim_{r_t \downarrow \lambda-\gamma} \frac{1 - \lambda r_t/(r_t+\gamma) - \gamma}{1 - \lambda/(r_t+\gamma)} = \infty > 1.$$

Thus, the statement in the lemma follows from the Intermediate Value Theorem. *QED*

Indeed, an explicit solution for $\widetilde{r}$ is obtained: $\widetilde{r} = \widetilde{r}(\lambda, \gamma) = (\lambda - \gamma^2)/(\lambda + \gamma)$. This lemma shows that for a sufficiently low interest rate, there are no self-financing entrepreneurs in the economy. This result might at first seem odd, but the reason is clear: under intermediated lending, an entrepreneur can borrow more than under direct lending, and this effect is greater as the interest rate becomes lower. Summarizing the argument so far, I obtain the following proposition, which states that a household's optimal financial decisions, given the interest rate $r_t$ and the inherited level of wealth $a_t$.

**Proposition 2.** *Suppose that (a)* $\lambda - \gamma < r_t \leq \widetilde{r}$*. Then, a household does not become an entrepreneur if his wealth is* $a_t \in [0, \underline{a}(r_t))$*, but he borrows from an FI to become an entrepreneur if his wealth is* $a_t \geq \underline{a}(r_t)$*. Next, suppose that (b)* $\widetilde{r} < r_t \leq 1-\gamma$*. Then, a household does not become an entrepreneur if his wealth is* $a_t \in [0, \underline{a}(r_t))$*, but he borrows from an FI if his wealth is* $a_t \in [\underline{a}(r_t), q)$ *or self-finances if* $a_t \geq q$*.*

Based on this proposition, one can derive what kind of income inequality arises in this economy, which is summarized in the following corollary.

**Corollary 1.** *In equilibrium, a household with* $a_t < \underline{a}(r_t)$ *earns the interest proceed* $r_t$ *from his savings, and one with* $a_t \geq \underline{a}(r_t)$ *earns the return one, which is greater than* $r_t$*, from the project.*

Note that the total revenue of an entrepreneur depends on whether he has self-financed or borrowed from an FI, and his inherited wealth level $a_t$. As in the case of no financial intermediation, the GINI index for income inequality can be calculated:

$$IncomeGINI_t(r_t) = \frac{(1-r_t)G_t(\underline{a}(r_t))[1-G_t(\underline{a}(r_t))]}{1-(1-r_t)G_t(\underline{a}(r_t))}.$$

If $G_t(q) > G_t(\underline{a}(r_t))$ and $G_t(q)(1-G_t(q)) > G_t(\underline{a}(r_t))[1-G_t(\underline{a}(r_t))]$, then $IncomeGINI_t^{NFI} > IncomeGINI_t(r_t)$.

## 3.5 Existence and Uniqueness of the Equilibrium Interest Rate for a Fixed Wealth Distribution

Now, I verify the existence of the equilibrium interest rate $r_t$ in period $t$, given the wealth distribution $G_t$. First, I look at the demand side of capital (i.e. the economy's total investment). In the case of $\widetilde{r} < r_t < 1-\gamma$, the aggregate demand for intermediated capital is

$$D^I(r_t) = \frac{1}{1-\lambda/(r_t+\gamma)}\int_{\underline{a}(r_t)}^{q} adG_t(a),$$

while the aggregate amount of self-financing is

$$D^S(r_t) = \int_q^{\infty} adG_t(a)$$

so that the aggregate demand for capital is $D(r_t) = D^I(r_t) + D^S(r_t)$ for $r_t \in (\widetilde{r}, 1-\gamma)$. Note that for a fixed $G_t$, $D^S(r_t)$ is indeed a constant. For this range, $D(r_t)$ is continuous, and decreasing in $r_t$. It is not strictly decreasing, though, since there can be a positive mass on $\underline{a}(r_t)$ or on $q$. These properties also hold for the case of $\lambda - \gamma < r_t < \widetilde{r}$, where

$$D(r_t) = \frac{1}{1 - \lambda/(r_t + \gamma)} \int_{\underline{a}(r_t)}^{\infty} a dG_t(a)$$

since there is no self-financing.

Now consider the case of $r_t = \widetilde{r}$. In this case, households with $a_t \geq \underline{a}(\widetilde{r})$ are indifferent between borrowing self-financing and borrowing from an FI. So, the demand for intermediated capital is between zero and

$$\frac{1}{1 - \lambda/(\widetilde{r} + \gamma)} \int_{\underline{a}(\widetilde{r})}^{\infty} a dG_t(a)$$

so that the aggregate aggregate demand for capital $D(r_t)$ is continuous at $r_t = \widetilde{r}$.

Last, consider the case of $r_t = 1 - \gamma$. As explained above, households with $a_t \in [\underline{a}(1 - \gamma), q)$ are indifferent between becoming an entrepreneur (by borrowing from an FI) and choosing $k_t = 0$. Due to the borrowing constraint (3), household $a_t \in [\underline{a}(1 - \gamma), q)$ can borrow up to $k_t = a_t$. Thus, the demand for intermediated capital when $r_t = 1 - \gamma$ is a correspondence:

$$D(1 - \gamma) = \left[0, \frac{1}{1 - \lambda} \int_{1-\lambda}^{q} a dG_t(a)\right].$$

It can be verified that $D(r_t)$ is continuous at $r_t = 1 - \gamma$. Overall, the aggregate demand is a continuous function of $r_t$ in the relevant range.

Next, I turn my attention to the supply side of capital. For any $r_t \in (\lambda - \gamma, 1 - \gamma]$, the aggregate supply of capital (i.e. the economy's total savings) is

$$K_t = \int_0^{\infty} a dG_t(a),$$

where $\int$ denotes Lebesgue integral. Note that it does not depend on $r_t$. It is, however, endogenously determined since it depends on $G_t$. The equilibrium interest rate in period $t$ is determined by the usual market clearing condition: $K_t = D(r_t)$. It is immediate to see the following proposition holds.

**Proposition 3.** *The equilibrium interest rate* $r_t \in (\lambda - \gamma, 1 - \gamma]$ *exists and is unique.*

### 3.6 Equilibrium Dynamics of the Wealth Distribution and the Interest Rate

Given the initial wealth distribution $G_0$, the market clearing condition also has a role to recursively determine the dynamics of the equilibrium interest rate $\{r_t\}_{t=0}^{\infty}$ together with the wealth dynamics $\{G_t\}_{t=1}^{\infty}$ (which is caused by dynastic motivation explained below). Here I assume that expectations are fully rational (or, players have perfect foresight since there is no uncertainty). First, take any period $t$, and suppose that $\widetilde{r} < r_t < 1-\gamma$. Then, the wealth dynamics in this case is described by

$$a_{t+1} = a_{t+1}(a_t; r_t) = \begin{cases} \beta(1+a_t) & \text{for } a_t \geq q, \\ \beta\left[1 + \dfrac{1-\lambda r_t/(r_t+\gamma)-\gamma}{1-\lambda/(r_t+\gamma)} a_t\right] & \text{for } a_t \in [\underline{a}(r_t), q), \\ \beta(1+r_t a_t) & \text{for } a_t \in [0, \underline{a}(r_t)), \end{cases}$$

where $\beta \in (0,1)$ is the parameter which describes how a household cares about his next generation; a household consumes fraction $(1-\beta)$ of his income, and leaves fraction $\beta$ to his son.[20] One may interpret this as an exogenous parameter of saving rate as in the Solow growth model.

Similarly, for $r_t \in (\lambda - \gamma, \widetilde{r}]$ the wealth dynamics is described by

$$a_{t+1} = a_{t+1}(a_t; r_t) = \begin{cases} \beta\left[1 + \dfrac{1-\lambda r_t/(r_t+\gamma)-\gamma}{1-\lambda/(r_t+\gamma)} a_t\right] & \text{for } a_t \geq \underline{a}(r_t), \\ \beta(1+r_t a_t) & \text{for } a_t \in [0, \underline{a}(r_t)). \end{cases}$$

Thus, for any equilibrium interest rate $r_t$, the wealth transition $a_t$ to $a_{t+1}$ is obtained. The wealth distribution dynamics $\{G_t\}_{t=1}^{\infty}$ is determined by the following

[20] This dynamics can be derived from generation $t$'s entrepreneur's utility maximization problem if we assume the following "warm-glow" utility function:

$$u = (1-\beta)\ln c + \beta \ln b,$$

where $c$ is the amount of his own consumption and $b$ is that of his bequest to his son. The indirect utility as a function of the realized income $I$ becomes

$$u(I) = (1-\beta)^{1-\beta}\beta^{\beta} I,$$

which is linear, so this formulation is consistent with the assumption that households are risk-neutral.

law of motion:

$$G_t = \int a_t(a_{t-1}; r_{t-1}) dG_{t-1}.$$

Instead of investigating the dynamics per se, I focus on the steady-state to study which financial pattern arises as well as the wealth distribution in the economy. Note that the limit wealth distribution $G_\infty(a)$ will have positive mass on fixed points of mapping $a_{t+1} = a_{t+1}(a_t; r_\infty)$, where $r_\infty$ is a limit interest rate. Thus, my goal is to analyze the properties of these fixed points. As is seen in the next section, the limit wealth distribution $G_\infty$ includes a finite number of mass points. While it does not resemble any conceivable wealth distribution in the real world, one can obtain interesting insights about the relationship between finance and inequality. For example, the steady-state GINI index of wealth can be analytically computed.

# 4 Steady-State Analysis

In this section, I study the relationship between inequality and financial intermediation in the steady state. First of all, it is immediate to see that if there were no enforcement problem, one would have $r_\infty = 1$ from the beginning of the economy, and in the steady state all households would have the same wealth level $a_\infty = \beta/(1-\beta)$, provided that $\beta < 1$. In each period, the income of all households is $1 + \beta/(1-\beta) = 1/(1-\beta)$. This is because for any period $t$ the wealth dynamics is described by $a_{t+1} = \beta(1 + a_t)$, and the steady-state wealth level is the fixed point of this mapping. Since there is no heterogeneity among households with respect to production ability or consumption preferences, the difference in the wealth level eventually vanishes. There would be no room for FIs from the beginning, and there would be no income or wealth inequality in the long run. These arguments are summarized in the following proposition.

**Proposition 4.** *There is no income or wealth inequality across households in the long run if there are no enforcement problems (the GINI indices for income and wealth inequalities are zero in the long run). That is, the wealth level of any* *household converges to* $\beta/(1-\beta)$, *and the income level of any household to* $1/(1-\beta)$ *in the long run.*

## 4.1 Steady-State without FIs

Before going to analyze the roles of financial intermediation, I check the steady state without FIs. Recall that in each $t$ households with $a_t < q$ has no way to use the backyard storage technology, while those with $a_t \geq q$ can invest in the project. Thus, the wealth dynamics is described by

$$a_{t+1} = \begin{cases} \beta(1+a_t) & \text{for } a_t \geq q, \\ \beta(1+\rho a_t) & \text{for } a_t \in [0,q). \end{cases}$$

I assume that the fixed cost for production is not large and that the return of the storage technology is not large either. Specifically, I make the following assumption to ensure the existence of the steady state where both rich households (entrepreneurs) and poor households (lenders) coexist in the long run. Then the wealth dynamics is described as in Figure 1.

**Assumption 3.** $2\beta \geq q > \beta(1+\rho)$.

Note that because $q < 2$ (see Subsection 3.1), the first part of the assumption imposes a restriction on the range of the warm-glow parameter: $\beta \in [q/2, 1)$. Note also that since $q \geq 1$ it must be the case that $\beta \geq 1/2$. Together with $\beta\rho < 1$ (from Assumption 2), I have $0 \leq \rho < \min\{\lambda-\gamma, 1/\beta, q/\beta-1\}$, and indeed $\sup_{\beta,q}\rho = \lambda-\gamma$.

It is seen that there are two fixed points of the mapping, $\beta/(1-\beta)$ for richer households, and $\beta/(1-\beta\rho)$ for poorer households. Notice that the initial wealth distribution $G_0$ completely determines the destiny of a household: the wealth of those with $a_0 < q$ (the fraction of which is $G_0(q)$) converges to $\beta/(1-\beta\rho)$, and that of those with $a_0 \geq q$ (the fraction of which is $1-G_0(q)$) to $\beta/(1-\beta)$. Unless $G_0(q) = 0$ or $G_0(q) = 1$, inequality necessarily arises.

Let the aggregate national wealth and the GINI index for wealth inequality in the steady state by denoted by $NW_{\infty}^{NFI}$ and by $WealthGINI_{\infty}^{NFI}$, respectively.

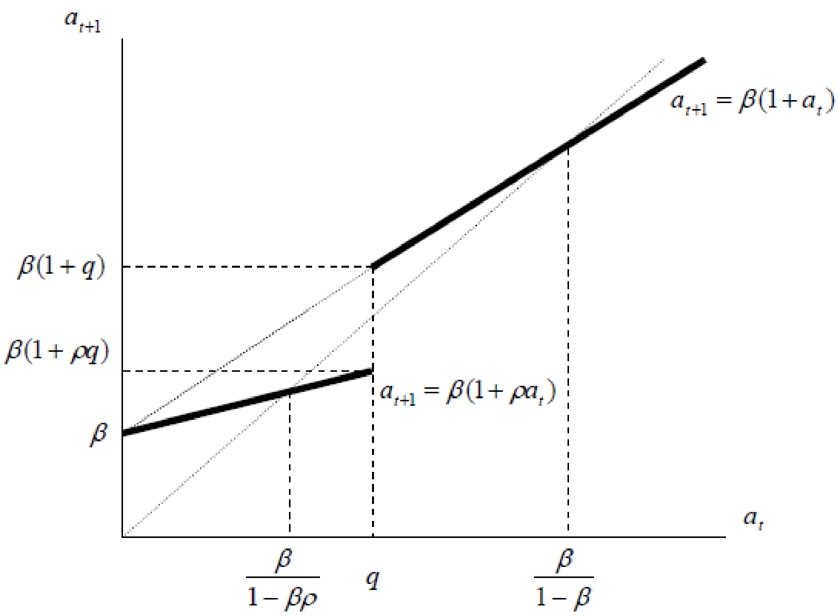


Figure 1: Wealth dynamics when there are no FIs.

Then, it is verified that (note that $\beta\rho < 1$ from Assumption 2):

$$NW_{\infty}^{NFI} = \beta\frac{(1-\beta)G_0(q) + (1-\beta\rho)[1-G_0(q)]}{(1-\beta)(1-\beta\rho)}$$

and

$$\begin{aligned} WealthGINI_{\infty}^{NFI} &= \frac{\beta(1-\rho)G_0(q)[1-G_0(q)]}{(1-\beta)G_0(q) + (1-\beta\rho)[1-G_0(q)]} \\ &= \frac{\beta^2(1-\rho)G_0(q)[1-G_0(q)]}{(1-\beta)(1-\beta\rho)NW_{\infty}^{NFI}}. \end{aligned}$$

As is expected, it is seen that $\partial NW_{\infty}^{NFI}/\partial\rho > 0$. I also have $\partial NW_{\infty}^{NFI}/\partial G_0(q) < 0$, though the sign of $\partial WealthGINI_{\infty}^{NFI}/\partial G_0(q)$ is indeterminate. If $G_0(q) > 1/2$, then $\partial WealthGINI_{\infty}^{NFI}/\partial G_0(q) > 0$, and for $\partial WealthGINI_{\infty}^{NFI}/\partial G_0(q) < 0$ to hold, it is necessary that $G_0(q) < 1/2$. This means that if there are many agents whose initial wealth is smaller than $q$ (so that $G_0(q) > 1/2$), then an increase in $G_0(q)$ worsens the inequality. In order to see the operation of this benchmark case, consider the following numerical example.

**Example 1.** Suppose that $\beta = 3/4$, $\rho = 4/15$, and $q = 1$. Then, we have $NW_{\infty}^{NFI}(\beta = 3/4) = 3 - 33G_0(q)/16$ and $WealthGINI_{\infty}^{NFI}(\beta = 3/4) = 33G_0(q)[1-$

$G_0(q)]/16NW_\infty^{NFI}(\beta = 3/4)$. Now, suppose that $\beta = 7/8$ (and $\rho = 4/15$, $q = 1$). In this case, $NW_\infty^{NFI}(\beta = 7/8) = 7 - 77G_0(q)/120$ and $WealthGINI_\infty^{NFI}(\beta = 7/8) = 539G_0(q)[1 - G_0(q)]/92NW_\infty^{NFI}(\beta = 7/8)$. We can verify that $7 - 77G_0(q)/120 > 3 - 33G_0(q)/16$ and $WealthGINI_\infty^{NFI}(\beta = 3/4) > WealthGINI_\infty^{NFI}(\beta = 7/8)$ for any $G_0(q) \in (0, 1)$. Thus, in this case, the higher $\beta$ is, the higher the national wealth and the less severe the inequality is.

## 4.2 Financial Intermediation and Inequality

Now, I consider the role financial intermediation in the wealth dynamics. Recall that there are five parameters; (i) $\lambda \in [0, 1)$; the effect of monitoring, (ii) $\gamma > 0$; the unit cost of monitoring, (iii) $\beta$; the "warm-glow" parameter, (iv) $q$; the fixed cost for production, and (v) $\rho$; the per-unit return of the backyard storage technology. As is seen below, $\rho$ does not appear in the case of active FIs because of Assumption 2. It is verified that the original Kuznets hypothesis does not hold; inequality persists in any steady state equilibria for any initial distribution $G_0$. This is an important feature because many existing models cannot exclude perfect equality out of the equilibria.[21] For comparison with the case of no FIs, I maintain the first part of Assumption 3 (i.e., $2\beta \geq q$) as well as Assumption 2 (i.e., $\lambda > \gamma + \rho$).

For notational convenience, I define the following function:

$$H(r_\infty) = 1 - \frac{\lambda r_\infty}{r_\infty + \gamma} - \gamma,$$

which is equal to $(1 - \gamma - r_\infty) + r_\infty \underline{a}(r_\infty)/q$ so that $H(r_\infty) > 0$ because $1 - \gamma > r_\infty$.

There are three possible fixed points of the mapping $a_{t+1} = a_{t+1}(a_t; r_\infty)$, which are

$$\begin{cases} a^*(r_\infty) = \dfrac{\beta}{1 - \beta r_\infty}, \\[2ex] a^{**}(r_\infty) = \left(1 - \beta q \dfrac{H(r_\infty)}{\underline{a}(r_\infty)}\right)^{-1} \beta, \text{ and} \\[2ex] a_\infty^{***} = \dfrac{\beta}{1 - \beta}, \end{cases}$$

[21] An exception is Mookherjee and Ray (2003).

where all the households who cannot borrow in the steady-state have wealth $a^*(r_\infty)$, those who borrow from an FI have $a^{**}(r_\infty)$, and those who self-finance have $a_\infty^{***}$.

Now consider the relationship among $a^*(r_\infty)$, $a^{**}(r_\infty)$, and $a_\infty^{***}$. Note that $a_\infty^{***} > a^*(r_\infty)$ for any $r_\infty \in (\lambda - \gamma, 1 - \gamma]$, $a^*(r_\infty) \gtreqless a^{**}(r_\infty) \Leftrightarrow r_\infty \gtreqless 1 - \gamma$, and $a_\infty^{***} \gtreqless a^{**}(r_\infty) \Leftrightarrow r_\infty \gtreqless \widetilde{r}(\lambda, \gamma)$. Note also that the domain of these functions is extended to $(\lambda - \gamma, 1]$ for the purpose of drawing the lines in the figure, though $r_\infty$ is indeed no greater than $1 - \gamma$. For all $r_\infty \in (\lambda - \gamma, 1 - \gamma]$, it is verified that

$$\begin{cases} \dfrac{\partial a^*(r_\infty)}{\partial r_\infty} = \dfrac{\beta^2}{(1 - \beta r_\infty)^2} > 0 \\ \\ \dfrac{\partial^2 a^*(r_\infty)}{\partial r_\infty^2} = \dfrac{2\beta^3}{(1 - \beta r_\infty)^3} > 0, \end{cases}$$

so that $a^*(r_\infty)$ is strictly increasing and convex. Note that $a^*(r_\infty)$ is bounded above zero because $\lim_{r_\infty \downarrow \lambda - \gamma} a^*(r_\infty) = \beta/[1 - \beta(\lambda - \gamma)]$ and $\lambda - \gamma < 1$. It is also verified that

$$\frac{\partial a^{**}(r_\infty)}{\partial r_\infty} = -\beta^2 q\lambda \frac{qH(r_\infty) + \gamma \underline{a}(r_\infty)}{(r_\infty + \gamma)^2[\underline{a}(r_\infty) - \beta q H(r_\infty)]^2} < 0$$

so that $a^{**}(r_\infty)$ is strictly decreasing. One can see that $\lim_{r_\infty \downarrow \lambda - \gamma} a^{**}(r_\infty) = \infty$, which comes from the fact that for any $a_t > 0$, $\overline{k}(a_t, r_t) \to \infty$ as $r_\infty \downarrow \lambda - \gamma$; entrepreneurs want to borrow infinitely if there is no borrowing limit because of the linearity of the production technology. This also implies that the limit interest rate is never equal to $\lambda - \gamma$ because that is inconsistent with the definition of steady state. Since we have $\lambda - \gamma < r_t \leq 1 - \gamma$ in any period $t$, we know that $\lambda - \gamma < r_\infty \leq 1 - \gamma$ in any steady state. Figure 2 is a graphical summary of the arguments so far. This figure is useful to investigate the existence of steady-state equilibria in the following analysis. From this figure, it is immediate to obtain the following proposition.

**Proposition 5.** *Perfect equality never arises and financial intermediation never disappears in any steady state unless* $G_0(q) = 1$ *or* $G_0(q) = 0$.

This result stands in contrast to the existing literature which allows perfect equality to arise as one of the equilibria. To see why this result holds in our model,

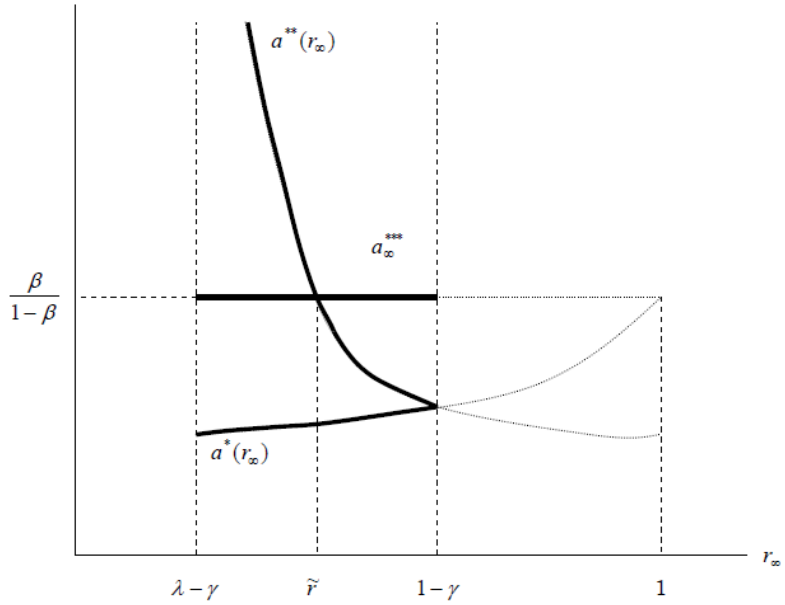


Figure 2: Relationship among $a^*(r_\infty)$, $a^{**}(r_\infty)$ and $a_\infty^{***}$.

note that perfect equality requires $r_\infty = 1$. In this case, the wealth dynamics should be $a_{t+1} = \beta(1 + a_t)$ for all households. This should mean that provided that $\beta/(1-\beta) \geq q$, all households become an entrepreneur and have the same amount of wealth, $\beta/(1-\beta)$. In this case, however, all entrepreneurs should rely on self-finance so that no FIs survive, which means that the capital market does not clear.

Next, observe a higher interest rate benefits net lenders. This is because given that they cannot borrow from anywhere the only source of poor households' income is lending. Note also that

$$\frac{\partial a^{**}(r_\infty)}{\partial \lambda} = \frac{\beta^2 q^2 (1-\gamma-r_\infty)}{[\underline{a}(r_\infty) - \beta q H(r_\infty)]^2 (r_\infty + \gamma)} > 0,$$

which means that the better the loan enforcement is the greater the entrepreneur's wealth is, while a change in $\lambda$ does not affect the lender's income. Recall that *given* the wealth distribution a change in $\lambda$ alleviates inequality by mitigating the borrowing constraint since $\partial \underline{a}(r_\infty)/\partial\lambda = -1/(r_\infty + \gamma) < 0$.

In what follows, I study a steady-state equilibrium, and consider the implication for inequality in the economy.

#### 4.2.1 Equilibria where All Financing is Intermediated

First, I consider the case of $\lambda - \gamma < r_\infty \leq \widetilde{r}$. First, for the steady-state to exist, one needs

$$\beta q \frac{H(r_\infty)}{\underline{a}(r_\infty)} < 1,$$

which can be written as

$$r_\infty > \frac{\beta\gamma(1-\gamma) + \lambda - \gamma}{1 - \beta + \beta(\lambda + \gamma)} \equiv \widehat{r}(\lambda, \beta, \gamma).$$

Otherwise, there is no steady-state because the wealth of entrepreneurs does not converge to a certain level. If this is not the case, then there are two fixed points, $a^*_\infty$ and $a^{**}_\infty$, with there being no capital market if the following is satisfied:

$$a^*(r_\infty) < \underline{a}(r_\infty) \leq a^{**}(r_\infty).$$

Here, it can be verified that $\widehat{r}(\lambda, \beta, \gamma) < \widetilde{r}(\lambda, \gamma)$ as long as $\beta < 1$. In order to ensure the existence of steady-state equilibria with $r_\infty \in (\lambda - \gamma, \widetilde{r}]$ (that is, for some $r_\infty \in (\lambda - \gamma, \widetilde{r}]$ that satisfies $a^*(r_\infty) < \underline{a}(r_\infty) \leq a^{**}(r_\infty)$ to exist), the following condition is necessary:

$$\underline{a}(\widehat{r}(\lambda, \beta, \gamma)) < a^{**}(\widehat{r}(\lambda, \beta, \gamma)).$$

This condition actually always holds as long as $\beta > 0$; it can be verified by noting that this condition is equivalent to $\underline{a}(\widehat{r}) - \beta q H(\widehat{r}) < \beta$ and that $\lambda/(\widehat{r} + \gamma) = [1 - \beta + \beta(\lambda + \gamma)]/(1 + \beta\gamma)$ holds

We have two cases for $r_\infty \in (\widehat{r}, \widetilde{r}]$ to consist of a steady state equilibrium: (i) when $\underline{a}(\widetilde{r}(\lambda, \gamma)) > a^*(\widetilde{r}(\lambda, \gamma))$ holds, and (ii) when $\underline{a}(\widetilde{r}) \leq a^*(\widetilde{r})$ and $\widehat{r} < r_L^+ \leq \widetilde{r}$, where $r_L^+$ is the larger solution of $a^*(r_\infty) = \underline{a}(r_\infty)$ if any. Note here that $a^*(r_\infty) = \underline{a}(r_\infty)$ is equivalent to

$$g(r_\infty) \equiv \beta r_\infty^2 - [1 - \beta/q + \beta(\lambda - \gamma)]r_\infty + \lambda - \gamma(1 - \beta/q) = 0,$$

which has at most two solutions. Note that $a^*(r_\infty) > \underline{a}(r_\infty) \Leftrightarrow g(r_\infty) > 0$.

In case (i), the equilibrium limit interest rate is $r_\infty \in (\max[\widehat{r}, r_L^-], \min[r_H^+, \widetilde{r}]]$, where $r_L^-$ is the smaller solution of $g(r_\infty) = 0$ and $r_H^+$ is a solution of $a^{**}(r_\infty) = \underline{a}(r_\infty)$, which is unique because $a^{**}(\cdot)$ is strictly decreasing and $\underline{a}(\cdot)$ is strictly increasing. Note that if $\underline{a}(\widetilde{r}) > a^*(\widetilde{r})$, then such an $r_L^- \in (\lambda - \gamma, \widetilde{r})$ exists by the Intermediate Value Theorem because and $\lim_{r_t \downarrow \lambda-\gamma} \underline{a}(r_t) = 0$, $\lim_{r_\infty \downarrow \lambda-\gamma} a^*(r_\infty)$, and the monotonicities of $\underline{a}(\cdot)$ and of $a^*(\cdot)$. It is verified that $r_L^+ \in (\widetilde{r}, 1/\beta)$ exists because $\underline{a}(\cdot)$ is bounded below $q$ and $\lim_{r_\infty \uparrow 1/\beta} a^*(r_\infty) = \infty$. Now, it is seen that $\underline{a}(\widetilde{r}) > a^*(\widetilde{r})$ is equivalent to

$$f(\lambda) \equiv (1-\beta)\lambda^2 + [\beta\gamma^2 + (1+\beta/q)\gamma + (1+1/q)\beta - 1]\lambda - \gamma[1-\beta/q+\beta\gamma(1-1/q)] < 0.$$

Here, it is verified that $f(1) = \beta(1+\gamma)^2/q > 0$, $f'(1) = (1-\beta) + \gamma(\beta\gamma + \beta/q + 1) + \beta/q > 0$, and $f(\gamma) = \gamma[\beta\gamma^2 + (1+2/q)\beta - 2(1-1/q)\beta\gamma - 2(1-\gamma)]$. Thus, letting $\lambda^+$ and $\lambda^-$ be defined by the larger and the smaller solution of $f(\lambda) = 0$, respectively,[22] one can verify that $f(\lambda) < 0$ if and only if $\max[\lambda^-, \gamma] < \lambda < \lambda^+$.

If case (ii) happens, then the equilibrium limit interest rate is $r_\infty \in (\max[\widehat{r}, r_L^-], r_L^+]$. For $r_L^+ \leq \widetilde{r}$ to exist, the determinant of $g(r_\infty) = 0$ should be positive, which implies

$$h(z) \equiv z^2 + 2\beta(\lambda+\gamma)z + \beta^2(\lambda-\gamma)^2 - 4\beta\lambda > 0,$$

where $z \equiv 1-\beta/q$ so that $1-\beta \leq z < 1-\beta/2$. Note that $h'(z) = 2z + 2\beta(\lambda+\gamma) > 0$ for $z \geq 1-\beta$. Thus, it is deduced that $h(z) > 0 \Leftrightarrow h(1-\beta) = (1-\beta)^2 + \beta^2(\lambda - \gamma)^2 - 2\beta[\lambda - \gamma + \beta(\lambda+\gamma)] > 0$.

Summarizing the argument above, I obtain the following proposition. Figure 3 illustrates one case of this equilibrium.

**Proposition 6.** *If* $\max[\lambda^-, \gamma] < \lambda < \lambda^+$ *or if*

$$(1-\beta)^2 + \beta^2(\lambda-\gamma)^2 - 2\beta[\lambda-\gamma+\beta(\lambda+\gamma)] > 0$$

[22] There always exist two solutions for $f(\lambda) = 0$ since the determinant is

$$[\beta\gamma^2 + (1+\beta/q)\gamma + (1+1/q)\beta - 1]^2 + 4\gamma(1-\beta)[(1-\beta/q) + \beta\gamma(1-1/q)],$$

which is positive because $1-\beta/q > 0$ and $q \geq 1$.

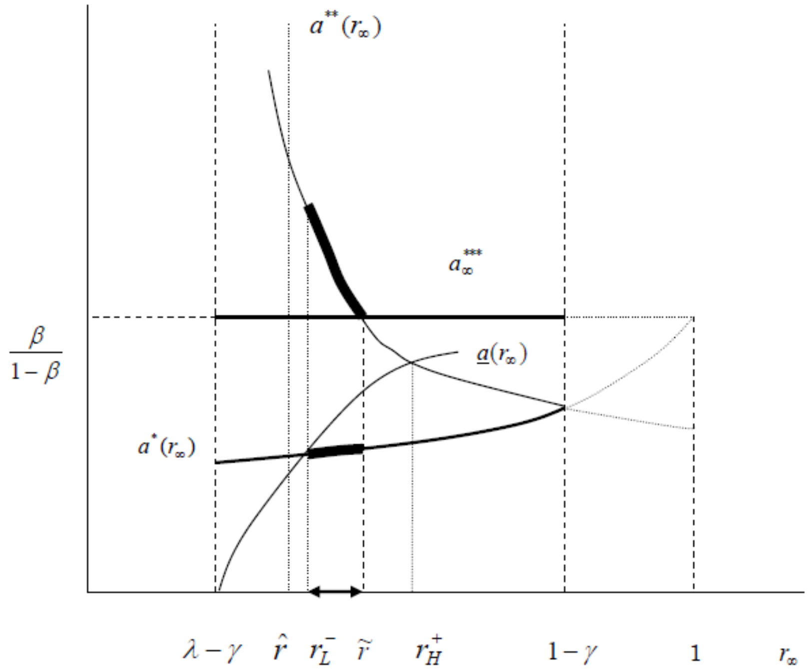


Figure 3: One case of the economy where all financing is intermediated.

*and* $\widehat{r}(\lambda,\beta,\gamma) < r_L^+(\lambda,\beta,\gamma,q)$*, then there exists a steady-state equilibrium with* $r_\infty \in (\max[r_L^-, \widehat{r}], \min[r_L^+, r_H^+, \widetilde{r}]]$*. In this equilibrium, the wealth of entrepreneurs is* $a^{**}(r_\infty)$*, and that of lending households is* $a^*(r_\infty)$*.*

Bold lines on the wealth levels and on the interest rate in Figure 3 depict a continuum of the steady-state equilibria where all entrepreneurs rely on financial intermediation and $r_\infty \in (r_L^-, \widetilde{r}]$.

Now, consider the capital market clearing condition. Letting $X_L(r_\infty)$ be the fraction of net lenders in the economy ("$L$" connotes the interest rate is low), I obtain

$$X_L(r_\infty)a^*(r_\infty) + [1 - X_L(r_\infty)]a^{**}(r_\infty) = [1 - X_L(r_\infty)]\frac{a^{**}(r_\infty)}{1 - \lambda/(r_\infty + \gamma)},$$

which implies

$$X_L(r_\infty) = \frac{\lambda(1 - \beta r_\infty)}{[1 - \beta(1-\gamma)](r_\infty + \gamma)}.$$

It can be shown that $\partial X_L(r_\infty)/\partial r_\infty = -\lambda(1+\beta\gamma)/[1-\beta(1-\gamma)](r_\infty+\gamma)^2 < 0$, which means that the greater the limit interest rate is, the less households are net lenders, which seems a natural consequence.

Let the aggregate wealth in the economy when $r_\infty \in (\lambda-\gamma, \widetilde{r}]$ be denoted by $NW_L(r_\infty)$. Then, it is shown that

$$NW_L(r_\infty) = \beta \frac{\lambda + r_\infty + \gamma}{[1-\beta(1-\gamma)](r_\infty+\gamma)}.$$

Similarly, the GINI index for wealth inequality in the steady state is given by

$$\begin{aligned} WealthGINI_L(r_\infty) &= 1 - X_L(r_\infty)\frac{X_L(r_\infty)a^*(r_\infty)}{NW_L(r_\infty)} \\ &\quad - \left(\frac{X_L(r_\infty)a^*(r_\infty)}{NW_L(r_\infty)} + 1\right)[1 - X_L(r_\infty)]] \\ &= \frac{X_L(r_\infty)[1-X_L(r_\infty)][a^{**}(r_\infty) - a^*(r_\infty)]}{NW_L(r_\infty)}. \end{aligned}$$

### 4.2.2 Equilibria where Not All Financing is Intermediated

Now, I consider the case of $\widetilde{r} < r_\infty \le 1-\gamma$. There are three possible fixed points, $a^*(r_\infty)$, $a^*(r_\infty)$ and $a_\infty^{***}$ if the following is satisfied:

$$a^*(r_\infty) < \underline{a}(r_\infty) \le a^{**}(r_\infty) < q \le a_\infty^{***}.$$

Thus, I need to ask whether there exists $r_\infty \in (\widetilde{r}, 1-\gamma]$ that satisfies the above relationship. First, it must be the case that

$$\beta/(1-\beta) \ge q,$$

which is rewritten as $\beta \ge q/(1+q)$. Recall now the restriction of the fixed cost ($1 \le q < 2$) and the first part of Assumption.3 ($\beta \ge q/2$). It is verified that $q/2 \ge q/(1+q)$. Thus, this condition actually always holds under the assumptions.

Since $a^*(r_\infty)$ is strictly increasing, there are two cases for $r_\infty \in (\widetilde{r}, 1-\gamma]$ to consist of a steady state equilibrium: (i) when $\underline{a}(1-\gamma) > a^*(1-\gamma) = a^{**}(1-\gamma)$ holds, and (ii) when $\underline{a}(1-\gamma) \le a^*(1-\gamma)$ and $\widetilde{r} < r_L^+ \le 1-\gamma$, where $r_L^+$ is the larger solution of $g(r_\infty) = 0$ if any.

In case (i), it is verified that $\underline{a}(1-\gamma) > a^*(1-\gamma)$ is equivalent to $\lambda < 1 - \beta/\{q[1-\beta(1-\gamma)]\}$, or $\beta < (1-\lambda)q/[1+(1-\gamma)(1-\lambda)q]$. The equilibrium limit interest rate is $r_\infty \in (\max[r_H^-, r_L^-], r_H^+]$, where $r_H^-$ is defined by

$$a^{**}(r_H^-) = q.$$

Recall that $r_H^+$ is defined by $a^{**}(r_H^+) = \underline{a}(r_H^+)$. Since $a^{**}(\cdot)$ is strictly decreasing, it is shown that $a^{**}(r_\infty) < q$ for $r_\infty > r_H^-$. It is also shown that $a^{**}(r_\infty) \geq \underline{a}(r_\infty)$ for $r_\infty \leq r_H^+$. Indeed, one can derive explicit forms of solutions:

$$\begin{cases} r_H^- = \dfrac{\beta\gamma(1/q-\gamma) + \lambda(1-\beta/q) - \gamma(1-\beta)}{1-(1+1/q)\beta + \beta(\lambda+\gamma)}, \text{ and} \\[2ex] r_H^+ = \dfrac{\beta\gamma(1+1/q-\gamma) + \lambda - \gamma}{1-(1+1/q)\beta + \beta(\lambda+\gamma)}, \end{cases}$$

where $r_H^- < r_H^+$ always holds.

If case (ii) happens, then the equilibrium limit interest rate is $r_\infty \in (\max[r_H^-, r_L^-], r_L^+]$. As in the last subsection, for $r_L^+ \leq \tilde{r}$ to exist, it must be the case that $(1-\beta)^2 + \beta^2(\lambda-\gamma)^2 - 2\beta[\lambda-\gamma+\beta(\lambda+\gamma)] > 0$. Summarizing the arguments so far, I obtain the following proposition.

**Proposition 8.** *If $q/2 \leq \beta < (1-\lambda)q/[1+(1-\gamma)(1-\lambda)q]$ or if $(1-\beta)^2 + \beta^2(\lambda-\gamma)^2 - 2\beta[\lambda-\gamma+\beta(\lambda+\gamma)] > 0$, then there exists a steady-state equilibrium with $r_\infty \in (\max[r_L^-, r_H^-], \min(r_L^+, r_H^+, 1-\gamma)]$. In this equilibrium, the wealth of entrepreneurs who self-finance is $a_\infty^{***}$, that of entrepreneurs who borrow from an FI is $a^{**}(r_\infty)$, and that of lending households is $a^*(r_\infty)$.*

Figure 4 illustrates one case of this equilibrium. Bold lines on the wealth levels and on the interest rate depict a continuum of the steady-state equilibria with $r_\infty \in (r_H^-, r_H^+]$.

Now, consider the capital market clearing condition. Letting $X_H(r_\infty)$ and $Y_H(r_\infty)$ be the fractions of net lenders and of entrepreneurs who rely on intermediated borrowing, respectively ("$H$" connotes the interest rate is high), I obtain

$$X_H(r_\infty)a^*(r_\infty) + Y_H(r_\infty)a^{**}(r_\infty) + [1 - X_H(r_\infty) - Y_H(r_\infty)]a_\infty^{***}$$

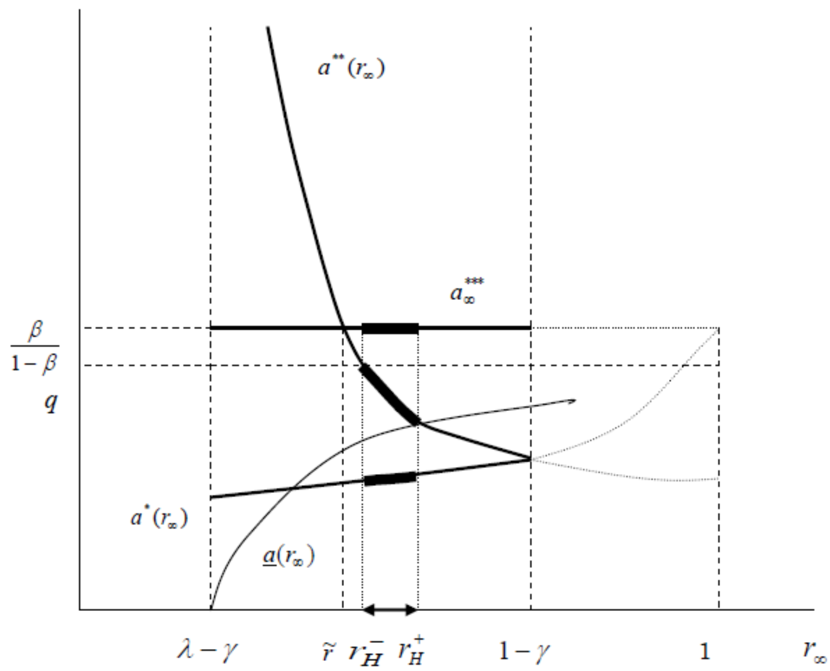


Figure 4: One case of the economy where not all financing is intermediated.

$$= \quad Y_H(r_\infty)\frac{a^{**}(r_\infty)}{1-\lambda/(r_\infty+\gamma)} + [1 - X_H(r_\infty) - Y_H(r_\infty)]a_\infty^{***},$$

which implies

$$\frac{\beta}{1-\beta r_\infty}X_H(r_\infty) = \frac{\beta\lambda}{[1-\beta+\beta(\lambda+\gamma)]r_\infty + [1-\beta(1-z)]\gamma - \lambda}Y_H(r_\infty).$$

Together with $X_H(r_\infty) + Y_H(r_\infty) = G_0(q)$, it is verified that

$$X_H(r_\infty) = \frac{\lambda G_0(q)(1-\beta r_\infty)}{[1-\beta(1-\gamma)](r_\infty+\gamma)}.$$

It can be shown that $\partial X_H(r_\infty)/\partial r_\infty = -\lambda G_0(q)(1+\beta\gamma)/[1-\beta(1-\gamma)](r_\infty+\gamma)^2 < 0$, which means that the greater the limit interest rate is, the less households are net lenders, which seems, again, a natural consequence.

Let the aggregate wealth in the economy when $r_\infty \in (\tilde{r}, 1-\gamma]$ be denoted by $NW_H(r_\infty)$. Then, I obtain

$$NW_H(r_\infty) \quad = \quad \left(1 - \frac{\lambda G_0(q)(1-\beta r_\infty)}{[1-\beta(1-\gamma)](r_\infty+\gamma)}\right)$$

$$\times \frac{\beta(r_\infty + \gamma)}{[1-\beta+\beta(\lambda+\gamma)]r_\infty + [1-\beta(1-\gamma)]\gamma - \lambda}$$
$$+\frac{\beta}{1-\beta}(1 - G_0(q)).$$

It is not easy, however, to determine whether $NW_H(r_\infty)$ is larger than $NW_L(r_\infty)$. Meanwhile, one can compare the fractions of lending households. It is verified that $X_L > X_H$ because it is equivalent to

$$\frac{\lambda(1-\beta r_\infty)}{[1-\beta(1-\gamma)](r_\infty+\gamma)} > \frac{\lambda G_0(q)(1-\beta r_\infty)}{[1-\beta(1-\gamma)](r_\infty+\gamma)},$$

where $r_\infty$ in the left hand side is $r_\infty \in (\lambda - \gamma, \widetilde{r}]$, and the one in the right hand side is $r_\infty \in (\widetilde{r}, 1-\gamma]$. Thus, more poor lending households are there in the equilibrium with $r_\infty \in (\lambda - \gamma, \widetilde{r}]$ than in the equilibrium with $r_\infty \in (\widetilde{r}, 1-\gamma]$. The GINI index for wealth inequality is given by

$$\begin{aligned} WealthGINI_H(r_\infty) &= \frac{X_H(r_\infty)Y_H(r_\infty)[a^{**}(r_\infty) - a^*(r_\infty)]}{NW_H(r_\infty)} \\ &\quad +[1 - X_H(r_\infty) - Y_H(r_\infty)] \\ &\quad \times \frac{X_H(r_\infty)(a^{***}_\infty - a^*(r_\infty)) + Y_H(r_\infty)(a^{***}_\infty - a^{**}(r_\infty))}{NW_H(r_\infty)}. \end{aligned}$$

Finally, notice that from Propositions 6 and 8, we know that if $\max[\lambda^-, \gamma] < \lambda < \lambda^+$ and $\beta > \max[q/2, (1-\lambda)/[\lambda + \gamma(1-\lambda)]$ (and Assumptions 2 and 3), there can exist two continua; one is with $r_\infty \in (\max[\widehat{r}, r_L^-], \min[\widetilde{r}, r_L^+]]$, and the other with $r_\infty \in (r_H^-, \min(r_H^+, 1-\gamma))$. Since $\beta > 1/2$, it is verified that $(2 - 1/\beta)$ lies between 0 and 1. This situation is depicted in Figure 5. Recall for a high monitoring effect ($\lambda \geq \lambda^+$), there does not exist a steady-state equilibrium with $r_\infty \in (\lambda - \gamma, \widetilde{r}]$ because it contradicts with the definition of the steady-state. Thus in this region, only $r_\infty \in (\max[\widehat{r}, r_L^-], \min[\widetilde{r}, r_L^+]]$ is permitted as a steady-state equilibrium. If the cost of monitoring is so low that $\gamma < 2 - \lambda - 1/\beta$, then the equilibrium with $r_\infty \in (r_H^-, \min(r_H^+, 1-\gamma))$ does not arise; all entrepreneurs want to borrow from an FI because the borrowing rate is low enough.

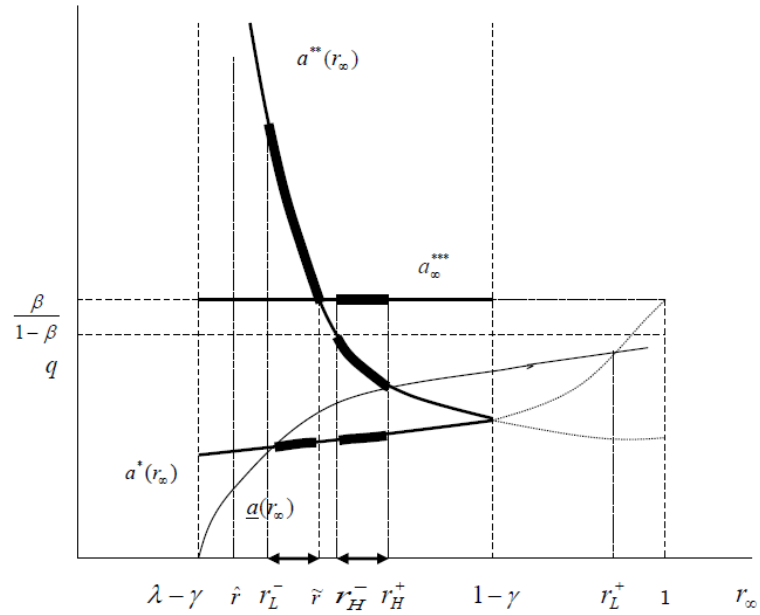


Figure 5: One case of two continua of multiple steady-state equilibria.

# 5 The Effects of Capital Account Liberalization on the Wealth Distribution

In this section, I consider the effects of capital account liberalization on the wealth distribution. By capital account liberalization (CAL), I mean the situation where agents in this economy get access to the international capital market populated with risk neutral foreign investors whose opportunity cost is $r^W > 0$. As such, agents in this economy takes this interest rate as given. I assume that CAL does not change the parameters of the model, and consider the steady-state of the economy after CAL is introduced.

First, suppose that the economy is in the steady-state with $r_\infty \in (\max[\widehat{r}, r_L^-], \min[r_L^+, r_L^+, \widetilde{r}]]$, and that the world interest rate is $r^W \in (\widehat{r}, r_\infty)$ (note that if $r^W < \widehat{r}$ then after CAL the economy does not go to a steady-state; borrowing entrepreneurs get infinitely wealthier). From Figure 3, it is easy to see that borrowing entrepreneurs support this regime change, while lending households are against it, as long as it does not make the threshold $\underline{a}(r_\infty)$ greater than $a^*(r_\infty)$. Note here that it is implicitly assumed that the FI sector still behaves competitively, so that they have to reduce the deposit rate to $r^W$ as well as the lending rate to $r^W + \gamma$. If majority voting is necessary for the regime change and $X_L(r_\infty) > 1/2$, then this economy does *not* implement CAL. However, if entrepreneurs can engage in lobbying, then CAL could be implemented. If the world interest rate is $r^W > r_\infty$, then entrepreneurs want to oppose to this regime change, while lending households welcome it.

Next, suppose that the economy is in the steady-state with $r_\infty \in (\max[r_L^-, r_H^-], \min[r_L^+, r_H^+, 1-\gamma]]$, and that the world interest rate is $r^W \in (r_H^-, r_\infty)$. From Figure 4, it is clear that entrepreneurs borrowing from an FI welcome CAL, lending households are against it, and self-financing entrepreneurs are neutral. However, if the world interest rate is $r^W \in (\max[\widehat{r}, r_L^-], \min[r_L^+, r_L^+, \widetilde{r}]]$, then all self-financing entrepreneur now support CAL because their successors (i.e., their sons) will eventually become richer. Whether CAL is implemented or not depends on the political decision-making system as well as the steady-state fractions of lending households and of entrepreneurs (self-finance or borrowing). Also, in this model, financial sectors, whether domestic or foreign, have a passive role because of the assumed perfect competition. If they are gaining rent due to the regulation or to the increasing return to scale of monitoring, then they will play a substantial role in the political decisions for CAL.

# 6 Concluding Remarks

In this paper, I have constructed a simple dynamic model to investigate the long-run relationship between types of financing (self-finance or intermediated finance) and the wealth distribution in an economy. Specifically, I have provided the steady-state characterization to see which finance pattern prevails in the economy as well as the characteristics of the wealth distribution. For any steady-state, the wealth inequality is higher and the interest rate is lower under the economy where intermediated capital is dominant. It is shown that for a wide range of parameters (concerning the benefit and the cost of monitoring) there are two continua of the steady-state equilibria; one is where all entrepreneurs rely on financial intermediation, and the other is where some borrowing entrepreneurs self-finance. The source of multiplicity is based on self-fulfilling prophecy; because the interest rate is low few richer borrowing entrepreneurs can borrow more, which makes the low interest rate is self-fulfilling due to the existence of many poor lending households and the opposite situation is also self-fulfilling. The effects of capital account liberalization on the wealth distribution was also discussed.

There still remains important questions unanswered in the present paper. In particular, how do financial crises affect the change in financial systems as a whole? How important are political processes for the evolution and changes in financial systems? Looking back at historical experiences (see, e.g., Allen and Gale (2000, Ch.2), Allen and Gale (2004a) and Bolton (2003)), we observe that it is often the case that characteristic features of financial systems change only if financial crises occur, regardless of whether they are due to domestic factors or to international pressure. It thus might suggest that financial crises have a positive effect on the working of an economy, of course with an inevitable turmoil, though. Analytical framework is necessary to investigate the issue of how political elements matter to the evolution and changes in financial systems (however, see, e.g., Bolton and Rosenthal (2002)).

Another interesting issue would be on the effect of international technological diffusion on the evolution and changes in domestic financial systems. What is the relationship between international technological diffusion and domestic evolution of financial systems? Based on their (1999) analysis, Allen and Gale (2000, Ch.13) suggest that the less innovative industry an economy has the bank-oriented system that economy adopts. However, domestic industrial progress and financial characteristics in one economy might be a result of international technological diffusion rather than of its original endowment. Naturally, this issue should be investigated with a full-fledged dynamic model. There has been an extensive literature investigating the issue of technological diffusion and economic growth. The literature, however, is silent on the differences in financial systems. These and other issues concerning the evolution and changes in financial systems are left for future research.

# References


Aghion, P., Bolton, P. (1997): 'A theory of trickle-down growth and development', *Review of Economic Studies*, 64 (2), pp. 151-172.

Alesina, A., Perotti, R. (1996): 'Income distribution, political instability, and investment', *European Economic Review*, 40 (6), pp. 1203-1228.

Allen, F., Gale, D. (1999): 'Diversity of Opinion and Financing of New Technologies', *Journal of Financial Intermediation*, 8 (1), 68-89.

Allen, F., Gale, D. (2000): *Comparing Financial Systems*, The MIT Press.

Allen, F., Gale, D. (2004): 'Competition and Financial Stability', *Journal of Money, Credit, and Banking*, 36 (3), pp. 453-480.

Ando, M., Yanagawa, N. (2004): 'Costs of enforcement in developing countries with credit market imperfections', Unpublished manuscript.

Banerjee, A. V., Newman, A. F. (1993): 'Occupational choice and the process of development', *Journal of Political Economy*, 101 (2), pp. 274-298.

Bolton, P. (2003): 'Toward a statutory approach to sovereign debt restructuring', IMF Working Papers 03/13.

Bolton, P., Rosenthal, H. (2002): 'Political intervention in debt contracts', *Journal of Political Economy*, 110 (5), 1103-1134.

Chakraborty, S., Ray, T. (2006): 'Bank-based versus market-based financial systems: a growth-theoretic analysis', *Journal of Monetary Economics*, 53 (2), pp. 329–350.

Chakraborty, S., Ray, T. (2007): 'The development and structure of financial systems', *Journal of Economic Dynamics and Control*, 31 (9), pp. 2920-2956.

Clarke, G., Xu, L. C., Zou, H. (2013): 'Finance and income inequality: test of alternative theories', *Annals of Economics and Finance*, 14 (2), pp. 493-510

Das, M., Mohapatra, S. (2003): 'Income inequality: the aftermath of stock market liberalization in emerging markets', *Journal of Empirical Finance,* 10 (1-2), pp. 217-248.

Deininger, K., Squire. L. (1998): 'New ways of looking at old ideas: inequality and growth', *Journal of Development Economics*, 57 (2), pp. 259-287.

Fajnzylber, P., Lederman, D., Loayza, N. (2002): 'What causes violent crime?', *European Economic Review*, 46 (7), pp. 1323-1357.

Freixas, X., Rochet, J.-C. (2008): *Microeconomics of Banking*, 2nd edition, The MIT Press.

Galor, O., Zeira, J. (1993): 'Income distribution and macroeconomics', *Review of Economic Studies*, 60 (1), pp. 35-52.

Galor,Oded, and Stelios Michalopoulos. 2006. "The Evolution of Entrepreneurial Spirit and the Process of Development." Mimeo, Brown University.

Greenwood, J., Jovanovic, B. (1990): 'Financial development, growth, and the distribution of income', *Journal of Political Economy*, 98 (5), pp. 1076-1107.

Greenwood, J., Sanchez, J. M., Wang, C. (2010): 'Financing development: the role of information costs', *American Economic Review*, 100 (4), pp. 1875-1891.

Greenwood, J., Smith, B. D. (1997): 'Financial markets in development, and the development of financial markets', *Journal of Economic Dynamics and Control*, 21 (1), pp. 145-181.

Holmström, B., Tirole, J. (1997): 'Financial intermediation, loanable funds, and the real sector', *Quarterly Journal of Economics* 112 (3), 663-691.

Iyigun, Murat F., and Ann L. Owen. 1999. "Entrepreneurs, Professionals, and Growth." *Journal of Economic Growth* **4** (2), 213-232.

Kiyotaki, N., Moore, J. (1997): 'Credit Cycles', *Journal of Political Economy*, 105 (2), pp. 211–248

Kuznets, S. (1955): 'Economic growth and income inequality', *American Economic Review*, 45 (1), pp. 1-28.

La Porta, R., Lopez-De-Silanes, F., Shleifer, A., Vishny, R. W. (1997): 'Legal Determinants of External Finance', *Journal of Finance*, 52 (3), pp. 1131-1150.

La Porta, R., Lopez-De-Silanes, F., Shleifer, A., Vishny, R. W. (1998): 'Law and Finance', *Journal of Political Economy*, 106 (6), pp. 1113-1155.

Li, H., Squire, l., Zou, H. (1998): 'Explaining international and intertemporal variations in income inequality', *Economic Journal*, 108 (446), pp. 26-43.

Matsuyama, K. (2000): 'Endogenous inequality', *Review of Economic Studies*, 67 (4), pp. 743-759.

Mookherjee, D., Ray, D. (2002): 'Contractual structure and wealth accumulation', *American Economic Review*, 92 (4), pp. 818-849.

Mookherjee, D., Ray, D. (2003): 'Persistent inequality', *Review of Economic Studies*, 70 (2), pp. 369-393.

Newman, A. F. (2007): 'Risk-bearing and entrepreneurship', *Journal of Economic Theory*, 137 (1), pp. 11-26.

Piketty, T. (1997): 'The dynamics of the wealth distribution and the interest rate with credit rationing', *Review of Economic Studies*, 64 (2), pp. 173-189.

Piketty, T. (2014): *Capital in the Twenty-First Century*, The Belknap Press of Harvard University Press.