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by

Mitsuyoshi Yanagihara
Akihiko Kaneko
Ai Sakai

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Golden Rule, Non-distortional Tax and Governmental Transfer

Mitsuyoshi Yanagihara[*] , Akihiko Kaneko[**] and Ai Sakai[***]

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Abstract

We consider the combination of non-distortional taxes (subsidies), a consumption tax and a wage income tax, in an overlapping-generations model to investigate how the rates of these taxes should be set to achieve a golden rule level of capital stock. We prove that if the initial steady state level of capital is below (above) the golden rule, the wage income tax rate should be negative (positive) and the consumption tax rate should be positive (negative) both at the steady state and along with the transitional path to the new steady state. We then show that, in transition, individuals can obtain benefit (loss) of a positive (negative) net transfer from (to) the government. In proving, a capital market equilibrium condition is necessary.

Keywords: Golden rule; Capital accumulation; Overlapping-generations model; non-distortional taxes

JEL classification: D91; E22; E62; H21

[*] Graduate School of Economics, Nagoya University; Furo-cho, Chikusa-ku, Nagoya, Aichi, 464-8601, Japan; Tel.: +81-52-789-5952; Fax: +81-52-789-4924; E-mail: yanagi@soec.nagoya-u.ac.jp.

[**] School of Political Science and Economics, Waseda University; 1-6-1 Nishi-waseda, Shinjuku-ku, Tokyo, 169-8050, Japan; Tel. & Fax: +81-3-3208-1197; E-mail: akaneko@waseda.jp.

[***] Graduate School of Economics, Nagoya University, Nagoya, Aichi, 464-8601, Japan; E-mail: sakai.ai@b.mbox.nagoya-u.ac.jp.

1. Introduction

We investigate what the combination of the rates of a consumption tax and a wage income tax can achieve a golden rule level of capital stock in an overlapping-generations (OLG) model.

It is no need to mention that, in theory, non-distortional taxes as represented by a lump-sum tax can lead to the first best situation of the economy. This is supported by the second theorem of welfare economics that "any Pareto-efficient allocation can be made a competitive equilibrium" (Hindriks and Miles, 2006: p.35). In other words, such adequate redistribution policies as non-distortional taxes can lead a targeted Pareto-efficient allocation. In the context of the OLG models, the golden rule level of capital can be achieved by utilizing non-distortional taxes to bring about redistribution between different generations.[1]

De la Croix and Michel (2002: p.129) proves that, in the neighborhood of the steady state, the direction of the optimal intergenerational lump-sum transfer is from the old (young) to the young (old) if the economy is dynamically efficient (inefficient) in an OLG model developed by Diamond (1965).[2] In proving this, they use the property of savings function, which is increasing in the income of the young and decreasing in that of the old. This property directly implies that the transfer from the old (young) to the

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[1] In reality, the lump-sum tax has not been prevailed. Rather, most countries have distortional taxes, e.g. a capital income tax and labor income tax which have distortional effects on the allocation of resource and/or time. Therefore, most of previous researches, whose pioneering works are Ramsey (1927) and Mirrless (1971), have mainly focused on how the second best taxation (distortional taxation) is under various economic circumstances and have little paid attention to what the first best taxation is.

[2] Ihori (2003: p.63) also mentions that the golden rule level of capital stock can be achieved by lump-sum transfers. Blanchard and Fischer (1989: p.110) also show that pay-as-you-go pension can be used for achieving the golden rule, as de la Croix and Michel (2002: p.150).

young (old) brings about an increase (decrease) in savings, which is required to raise (lower) the level of capital when the economy is dynamically efficient (inefficient). Thus, the interest of De la Croix and Michel (2002) is on the *direct* income transfer between individuals to control *savings*.

Besides the lump-sum transfer, in an overlapping-generations model, a wage income tax works as a non-distortional, lump-sum tax as long as a labor is inelastically supplied. In addition, a consumption tax can act as a non-distortional tax if its rate is common both for the young and the old generations. These imply that an adequate combination of these two kinds of taxes, in other words, the *indirect* income transfer via a consumption tax to control *the income of the young* can also lead the capital level of the economy to the golden rule.[3] However, surprisingly, this has never been proved formally. In real, the savings function approach developed by De la Croix and Michel (2002) cannot be applied to the case of consumption tax because it is *not* a direct income transfer.

This paper, therefore, proves how the wage income tax rate and the consumption tax rate are set to achieve a golden rule level of capital both in the steady state and along with the transitional path from the original steady state to the new steady state. For the consumption tax not to have a distortional effect on the individual's utility maximization, the consumption tax rate is common both for the young and the old in the same period. In proving, we use the condition that the economy monotonically converges to the steady state, which has its source from the capital market equilibrium condition. Roughly speaking, in contrast to De la Croix and Michel (2002) taking the

[3] The indirect income transfer via a combination of these taxes, as well as the direct income transfer, can make the income of the young more by sacrificing the resource owned by *the initial old* generation.

savings function approach, we take a dynamic stability condition approach.

We show that (i) if the initial steady state level of capital is below (above) the golden rule (the equilibrium of the economy is dynamically efficient), the wage income tax rate should be negative (positive) and the consumption tax rate should be positive (negative) both at the steady state and along with the transitional path in the neighborhood of the steady state; (ii) along with the transitional path, individuals can obtain benefit (loss) from a positive (negative) net transfer from (to) the government; (iii) once the new steady state equilibrium achieves the golden rule level of capital, the value of the net transfer becomes zero. These findings correspond to the ones shown in De la Croix and Michel (2002), so that our paper complements the argument of De la Croix and Michel (2002).

This paper is organized as follows: Section 2 presents a basic framework of this analysis just by reviewing the standard overlapping-generations model. Based on the model, Section 3 shows what the combination of consumption and wage income tax/subsidy both in the steady state and along with the transitional path to the steady state. Section 4 concludes.

2. Model

We use the Diamond's (1965) overlapping-generations model. The economy starts period 1 and lasts forever. Capital stock and the number of population in period 1, K_1 and L_1 are given and a population growth rate, $n \equiv \frac{L_{t+1} - L_t}{L_t} \geq -1$, is assumed to be constant over time.[4] In every period, there are two types of individuals, the young and the old.

2.1. Individuals

Individuals are identical and live for two periods: the young and the old periods. In period *t*, when they are young, they supply their labor inelastically and earn wages, w_t, either for their consumption in the young period, c_{1t}, or savings, s_t. As the government taxes both on their wages and on their consumption by tax rates τ_t^w and τ_t^c respectively, the budget constraint in their young period can be given by:

$$(1-\tau_t^w)w_t = (1+\tau_t^c)c_{1t} + s_t . \quad (1)$$

[5]

In period *t+1*, when they become old, they retire and just consume based on their savings in the previous period. As the same with consumption in their young period, the consumption in their old period is taxed by a rate of τ_{t+1}^c. Denoting the interest rate on the savings by r_{t+1}, the budget constraint in their old period is expressed as:

$$(1+r_{t+1})s_t = c_{2t+1} \quad (2)$$

Together with these two budget constraints (1) and (2) in each period, the lifetime budget constraint can be obtained as:

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4 Throughout this paper, lower case letters express per capita variables and larger ones do aggregate variables.

5 Here the tax rates are defined as positive (negative) if they are taxes (transfers from the government).

$$(1-\tau_t^w)w_t = (1+\tau_t^c)c_{1t} + \frac{(1+\tau_{t+1}^c)}{(1+r_{t+1})}c_{2t+1}\,. \tag{3}$$

The utility of individuals who are young in period *t* is defined as a function of c_{1t} and c_{2t+1}:

$$U_t \equiv U(c_{1t}, c_{2t+1}) \tag{4}$$

Finally, the individuals maximize the utility, (4), subject to the lifetime budget constraint (3). This gives consumption functions $c_{1t}(w_t, r_{t+1}; \tau_t^w, \tau_t^c, \tau_{t+1}^c)$ and $c_{2t+1}(w_t, r_{t+1}; \tau_t^w, \tau_t^c, \tau_{t+1}^c)$, leading to the savings function and an indirect utility function:

$$s_t = s(w_t, r_{t+1}; \tau_t^w, \tau_t^c, \tau_{t+1}^c) \tag{5}$$

$$\begin{aligned} V_t &\equiv U(c_{1t}(w_t, r_{t+1}; \tau_t^w, \tau_t^c, \tau_{t+1}^c), c_{2t+1}(w_t, r_{t+1}; \tau_t^w, \tau_t^c, \tau_{t+1}^c)) \\ &= V\,(w_t, r_{t+1}; \tau_t^w, \tau_t^c, \tau_{t+1}^c)\,. \end{aligned} \tag{6}$$

As in Blanchard and Fischer (1989), we assume (1) $0 < \partial s_t / \partial w_t < 1$ and (2) $0 < \partial s_t / \partial r_t$. The former means that a marginal propensity to save is between 0 and 1, implying that both consumption in the young and the old periods are normal. The latter, on the other hand, implies that the substitution effect dominates the income effect. In special, like the other previous literatures, assumption (2) has a critical role for the latter analysis.

2.2. Firms

Using capital, K_t, and labor, L_t, firms produces output. Aggregate production function is given as $Y_t = F(K_t, L_t)$. If it exhibits a constant returns to scale, then dividing both sides by L_t gives a per capita production function:

$$y_t = F(\frac{K_t}{L_t}, 1) \equiv f(k_t), \tag{7}$$

where y and k are the output and capital in per capita terms, respectively. Under a perfect competition, the first order conditions for profit maximization gives:

$$r_t = f'(k_t), \tag{8}$$

$$w_t = f(k_t) - k_t f'(k_t) \tag{9}$$

2.3.Government

In our analysis, we will put the following three assumptions for the government to achieve the golden rule level of capital stock.

Assumptions

(i) The government can use consumption and wage income taxes.

(ii) Balanced budget is kept: No government bond is permitted to be issued.

(iii) The rate of consumption tax in the young period and in the old period for every individual are set to be the same for the consumption tax to be non-distortional..

Assumption (iii), in turn, implies that the consumption tax rate should be kept constant over time: $\tau_t^c = \tau_{t+1}^c = \cdots = \tau_\infty^c \equiv \tau^c$. To the contrary, the wage income tax rate is variable so as to keep the budget balanced.

In sum, the government budget constraint can be written as:

$$\tau_t^w w_t(k_t) + \tau_t^c c_{1t}(w(k_t), r(k_{t+1}); \tau_t^w, \tau^c) + \frac{\tau_t^c}{(1+n)} c_{2t}(w(k_t), r(k_{t+1}); \tau_t^w, \tau^c) = 0\,. \quad (10)$$

2.4. Equilibrium

A capital market equilibrium is required for this economy to achieve a general equilibrium. To put it more concrete, in period t, by using (5), (8) and (9), capital market equilibrium condition can be written as:

$$k_{t+1} = \frac{s\ (w(k_t), r(k_{t+1}); \tau_t^w, \tau^c)}{1+n}\,, \quad (11)$$

which exhibits the dynamics of k_t and k_{t+1}, with dependent of the two tax rates.

3. Tax Rates for the Achievement of the Golden Rule

In this section, we will show when a combination of tax/subsidy on wage income/consumption along the transitional path achieves the golden rule level of capital stock in the steady state, how net transfer from/to the government becomes. Following

De la Croix and Michel (2002), we limit our argument on the transitional path in the neighborhood of the steady state.

3.1. Net Transfer in Young from the Government

Before investigating, from the government budget constraint (10), the following Lemma can be easily obtained.

Lemma 1. Under the Assumptions (i) to (iii), if $\tau_t^w > (<) 0$ and $\tau^c < (>) 0$, then,

$$\tau_t^w w_t(k_t) + \tau^c c_{1t}(w(k_t), r(k_{t+1}); \tau_t^w, \tau^c) > (<) 0 \text{ and } \frac{\tau^c}{(1+n)} c_{2t}(w(k_t), r(k_{t+1}); \tau_t^w, \tau^c) < (>) 0$$

necessarily holds.

This Lemma 1 means that if the wage income is subsidized (taxed), then the income of the young necessarily rises (falls). This is because the consumption tax rate is not assumed to be differentiated between the young and the old and only the young can work to earn wages. In other words, the absolute value of the tax on wage income cannot be exceeded by that on consumption in young.

Here consider the situation where the economy is dynamically efficient, $r > n$. Then, income should be subsidized by financing a tax on consumption, because the amount of savings, and therefore, capital, has to be more. As proved in De la Croix and Michel (2002), to save more, the income of the young should be more. Therefore, income should be transferred from the old to the young. Together with Lemma 1, the income level of the young can be raised by wage income subsidy, so that consumptions both of the young and the old should be taxed. The reverse also holds if the economy is dynamically inefficient.

3.2. Net Transfer from the Government through the Lifetime

Again we consider that the economy is dynamically efficient, namely, the level of capital is less than that of the golden rule. As we have seen in the previous subsection, the income of the young should be raised. In our framework here, it directly implies that the lifetime income should be raised. Therefore, in this subsection, we will confirm that individuals receive a benefit from the combination of the wage income subsidy and the consumption tax by inducing a positive net transfer from the government.

From (3) deducting (10),

$$w_t(k_t) = c_{1t}(w(k_t), r(k_{t+1}); \tau_t^w, \tau^c) + \frac{(1+\tau_{t+1}^c)}{(1+r_{t+1}(k_{t+1}))} c_{2t+1}(w(k_t), r(k_{t+1}); \tau_t^w, \tau^c)$$

$$- \frac{\tau_t^c}{(1+n)} c_{2t}(w(k_{t-1}), r(k_t); \tau_{t-1}^w, \tau^c) \qquad (12)$$

Noting $\tau_t^c = \tau_{t+1}^c = \cdots = \tau_\infty^c \equiv \tau^c$ and eliminating c_{2t} and c_{2t+1} by using (2), (12) can be rewritten as:

$$w(k_t) = c_{1t}(w(k_t), r(k_{t+1}); \tau_t^w, \tau^c) + \frac{1}{(1+r(k_{t+1}))} c_{2t+1}(w(k_t), r(k_{t+1}); \tau_t^w, \tau^c)$$

$$+\frac{\tau^c}{(1+\tau^c)}\{s(w(k_t),r(k_{t+1});\tau_t^w,\tau^c)-\frac{(1+r_{t+1}(k_{t+1}))}{(1+n)}s(w(k_{t-1}),r(k_t);\tau_{t-1}^w,\tau^c)\}$$

Then, using a capital market equilibrium condition (11), we can obtain:

$$w(k_t)-\frac{\tau^c}{(1+\tau^c)(1+n)}\{(1+n)k_{t+1}-(1+r(k_{t+1}))k_t\}$$

$$=c_{1t}(w(k_t),r(k_{t+1});\tau_t^w,\tau^c)+\frac{1}{(1+r(k_{t+1}))}c_{2t+1}(w(k_t),r(k_{t+1});\tau_t^w,\tau^c) \qquad (13)$$

The left hand side of (13) means the net lifetime income. Therefore, we will investigate whether its second term,

$$-\frac{\tau^c}{(1+\tau^c)}\{(1+n)k_{t+1}-(1+r(k_{t+1}))k_t\}, \qquad (14)$$

is positive or negative, in other words, exhibits the net transfer from individuals to the government (a tax) or that from the government to the individuals (a subsidy).

3.3. Dynamic Stability Condition

To see the condition that the per capita capital monotonically converges to the steady state, we firstly linearize the capital accumulation equation (12) around the neighborhood of the steady state:

$$k_{t+1} - k^* = \phi'(k_t - k^*) \tag{15}$$

where "*" represents the value of the variable evaluated in the steady state. Because $0 < \phi' < 1$ is the condition for the monotonic convergence, we can write this condition for the monotonic convergence as:

$$0 < k_{t+1} - k^* < k_t - k^* \tag{16}$$

3.4. Positive Net Transfer under Dynamic Efficiency

Finally, we prove that (14) is positive by using the condition for the monotonic convergence, (16), in the neighborhood of the steady state. At a glance, from the budget constraint of the government, (10), it seems to be natural that the lifetime income also increases in dynamic efficiency because, as De la Croix and Michel (2002) indicates, the income of the young should be raised for increasing the savings by subsidizing the wage income. However, the net lifetime income is possible to decrease, as long as the present value of the net amount of consumption tax is so large that it dominates the wage income subsidy.

Interestingly, the statement above mentioned *never* hold. To see this, we transform the inside of the braces in the left hand side of (14):

$$\begin{aligned}&(1+n)k_{t+1} - (1+r_{t+1})k_t \\ &= (1+n)(k_{t+1} - k^*) + (1+n)k^* - (1+r_{t+1})(k_t - k^*) - (1+r_{t+1})k^*\end{aligned}$$

$$= (1+n)(k_{t+1} - k^*) - (1+r_{t+1})(k_t - k^*) + (n - r_{t+1})k^* . \tag{17}$$

From (16) and $r > n$, it can be easily verify that (17) is necessarily negative. This, in turn, implies that the net transfer from the government to individuals, $-\frac{\tau^c}{(1+\tau^c)}\{(1+n)k_{t+1} - (1+r_{t+1})k_t\}$, is positive if and only if $0 < \tau^c$.

Similarly, we can see that $0 > \tau^c$ must hold along with the transitional path in the neighborhood of the steady state to achieve the golden rule level of capital under the dynamic inefficient. Therefore, we can here obtain the following Proposition:

Proposition 1

Consider the combination of tax (subsidy) of wage income (consumption), and subsidy (tax) of consumption (wage income) when the economy is dynamically efficient (inefficient). Then in the transitional path in the neighborhood of the steady state, net transfer from the government (the individuals) to the individuals (the government) is positive (negative).

It should be noted that to obtain this Proposition 1, we utilize the condition for monotonic convergence, in contrast to the proof given by De la Croix and Michel (2002). As De la Croix and Michel (2002) focuses on the relationship between the intergenerational lump-sum transfer and savings, it requires the feature of savings function. On the other hand, as our proof just focuses on the difference in the levels of capital in the neighborhood of the steady state, the condition for monotonic convergence of the capital to the steady state level is essential.

3.5. Zero Transfer in the Steady State

Along with the proof of Proposition 1 for transitional path, we can also have another Proposition regarding the amount of the transfer from/to the government in the steady state. Because $r = n$ holds in the golden rule and $k_{t+1} = k_t = k*$ hold in the steady state, the net transfer (14) becomes zero, or equivalently, (13) can be written as:

$$w* = c_1* + \frac{1}{(1+r*)} c_2* . \tag{18}$$

Therefore, we can finally have:

Proposition 2

Consider the combination of tax (subsidy) of wage income (consumption), and subsidy (tax) of consumption (wage income). When the economy achieves the golden rule level of capital, consumption should be taxed (subsidized) and wage income should be subsidized (taxed) if the economy starts from a dynamic efficient (inefficient) region. However, the net transfer from/to the government becomes zero in the steady state.

It is certain that even if the new steady state of the golden rule can be achieved, the combination of the signs of the wage income tax and the consumption tax in that steady state is the same with the one along with the transitional path from the initial steady state. However, in the new steady state, the net transfer from the government vanishes.

This also means that the net amount of transfer from the old to the young (or, from the young to the old) becomes nullified in the steady state of the golden rule.

4. Conclusion

We have shown what the combination of the rates of a consumption tax and a wage income tax can achieve a golden rule level of capital stock in an overlapping-generations model. We firstly confirmed that if the initial steady state level of capital is below (above) the golden rule, the wage income should be subsidized and the consumption should be taxed both at the steady state of the golden rule and along with the transitional path in the neighborhood of the steady state. Then, we could see that along with the transitional path, individuals can obtain benefit (loss) from a positive (negative) net transfer from (to) the government. Finally, in a new steady state of the golden rule, the value of the net transfer becomes zero.

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References

Olivier J. Blanchard and Stanley Fischer. *Lectures on Macroeconomics*, Cambridge, MA: The MIT Press, 1989.

De la Croix, David and Philippe Michel. *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*, Cambridge, U.K. : Cambridge University Press, 2002.

Diamond, Peter, A. “National Debt in a Neoclassical Growth Model,” *American Economic Review* 55 (No. 5 1965): 1126-1150.

Hindriks, Jean and Miles, Gareth, D. *Intermediate Public Economics*, Cambridge, Massachusetts: MIT Press, 2006.

Ihori, Toshihiro. *Theory of Taxiation* (*Kazei no Keizai Riron*, in Japanese), Tokyo, Japan, Iwanami Shoten, 2003.

Mirrless, James. “An Exploration in the Theory of Optimum Income Taxation,” *Review of Economic Studies* 38 (Vol. 38, No.2, 1971): 175-208.

Ramsey, Frank. “A Contribution to the Theory of Taxation,” *Economic Journal* 37 (Vol. 37, 1927): 47-61.